HETEROGENEOUS BELIEFS, COLLATERALIZATION, AND TRANSACTIONS IN GENERAL EQUILIBRIUM

A Dissertation

by

XU HU

Submitted to the Office of Graduate Studies of
Texas A&M University
in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY

August 2011

Major Subject: Economics

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ABSTRACT

Heterogeneous Beliefs, Collateralization, and Transactions

in General Equilibrium. (August 2011)

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Co–Chairs of Advisory Committee: Dr. Leonardo Auernheimer
Dr. Rajiv Sarin

This study includes two theoretical works. In both works, I assume that economic agents have heterogeneous beliefs. I study collateralized loan transactions among economic agents arising from the divergent beliefs. Moreover, I make collateral requirements endogenously determined, along with interest rates and loan quantities.

The theme of the first work is to study private transactions in currency crises. I assume that domestic residents have different beliefs on how resilient the central bank is in defending the currency. Due to the different beliefs, domestic residents willingly borrow and lend among themselves. I show that the heterogeneity of beliefs per se brings stability to the system, but that short-term collateralized loans among domestic residents arising from the divergent opinions make an exchange rate peg vulnerable.

The second work is to understand credit default swaps in general equilibrium. The model features a market for a risky asset, a market for loans collateralized by the risky asset, and a market for credit default swaps referencing these loans. I show that the introduction of credit default swaps only as insurance has no effect on the price of the risky asset. And the introduction of credit default swaps both as insurance and as tools for making side bets depresses the price of the risky asset in general but has no effect when the majority of the economy hold bearish views on the risky asset.

To Pepe

ACKNOWLEDGMENTS

I received enormous support and help from many people while writing this dissertation. I thank them all for their kindness. Below I would like to express my gratitude to the members of my advisory committee in particular.

Back in the spring of 2008, I was taking a research course under the guidance of Dr. Leonardo Auernheimer (Pepe). At that time, I was sure that I would love to do some theoretical work, but I was not sure what to write about in the dissertation. Pepe, generous as he always is, shared with me some of his thoughts on heterogeneous beliefs. He suggested to me two lines to pursue: one, to understand the linkage between the heterogeneity of beliefs and the volume of transactions in asset markets; and two, to investigate implications of heterogeneous beliefs in currency crises. Following his suggestions, I explored the first line for about a year, during which time I regularly presented my findings to Pepe and he returned his comments. Pepe was a wonderful listener. In our conversations, he always had the patience to let me finish talking even if he disagreed. Pepe was a good theorist. On many occasions, he brought me the idea that two things might appear to be totally different, but they in fact are the same thing when being viewed from a particular perspective. I guess that is the beauty of isomorphism.

In the summer of 2009, I started to think about taking heterogeneous beliefs to currency crises. Since then, many new questions started to unfold, which Pepe and I had not foreseen at the outset, such as the problems on collateral. My attempts to answer these questions led to the present work. On this road, Pepe worked with me until his last days. Even in the days when he was sick, Pepe kept coming to the office and chatted with me about my research. I guess that was where his passion lay, advising students and thinking.

Pepe not only cared if my dissertation proceeded well but he also cared about my personal life. In our regular meetings, Pepe often asked me if I had a good time in Austin with my friends, if my parents were doing well, or if I was getting better from my allergies. Pepe also often shared his family stories with me. Such a close tie with Pepe made the thesis-writing process full of joy.

Pepe taught me many lessons in economics and in other subjects too, such as English, from which I benefited tremendously. He was and will always be a role model for me, not only as a good economic theorist but also as a wonderful adviser and a great human being. My debt to him goes far beyond this present study.

Dr. Rajiv Sarin has also helped me a lot. Discussions with him have been very useful. I benefited from his suggestion to start with simple examples when I was stuck with the question of endogenizing collateral at the end of 2009. He also gave me useful comments on my unsuccessful presentation in the department seminar at the end of 2010. Since the spring of 2011, Dr. Rajiv Sarin met with me regularly to check the progress of the work. Without his guidance, my research would not have proceeded so well. I am very thankful for his help.

The completion of the present study is incomprehensible without the support and help from other committee members. I thank Dr. Thomas Saving for his interest in my work and his insightful comments. I thank Dr. David Bessler for his insightful observations and interesting questions. I thank Dr. Amy Glass for attending my presentation in the department seminar and her useful comments.

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CHAPTER I

INTRODUCTION

This study includes two theoretical works. They are unrelated in the subject matter, but they share several similarities from the modeling perspective. In both works, I assume that economic agents have heterogeneous beliefs. I study collateralized loan transactions among economic agents arising from divergent beliefs. Moreover, I make collateral requirements endogenously determined in the market for loans, along with interest rates and loan quantities. The first work is to study transactions within the private sector in the context of speculative currency crises. The second one is to understand credit default swaps in a general equilibrium model. Below, I introduce the subject, discuss key research questions and related works, and preview main findings briefly.

A. Speculative currency crises

A crisis of a pegged nominal exchange rate is an episode in which the public sell off domestic money in foreign exchange markets in a sizable scale, as the public's confidence in the exchange rate peg falters; meanwhile to defend the peg, the central bank intervenes, selling foreign exchanges and buying domestic money; as a result, the central bank loses foreign reserves in a short period of time; such a drastic decline in foreign reserves triggers the central bank to abandon the peg and devalue the currency.

Theoretical models in the literature of currency crises often assume the public to

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This dissertation follows the style of *Econometrica*.

be a homogeneous entity and ignore possible interactions within the private sector in time of crisis. First-generation models, á la, Krugman (1979), and Flood and Garber (1984), show that due to fiscal imbalances, a government inevitably abandons a fixed exchange rate at a predictable time. On the contrary, second-generation models, á la, Obstfeld (1986), show that crises can be unpredictable due to the existence of multiple equilibria, i.e., a fixed exchange rate can be brought down by self-fulfilling speculative attacks, even if it would have been viable perpetually in the absence of attacks.

One notable strand of research, departing from the paradigm of homogeneous agents, is pioneered by Morris and Shin (1998). They introduce private information into a second-generation model and show the uniqueness of equilibrium. A recent contribution by Broner (2008) introduces private information into a first-generation model. And he shows that multiple equilibria exist and that unpredictable and large devaluations are possible in certain equilibria.

Nevertheless, there is no theoretical attempt so far made in the literature to understand transactions within the private sector in the context of currency crises. In this work, I postulate that domestic residents have heterogeneous beliefs regarding how resilient the central bank is in defending the currency. And I study short-term collateralized loan transactions arising from the divergent beliefs among domestic residents.

In particular, I consider a small open economy with perfect capital mobility and a pegged nominal exchange rate. I assume that the domestic central bank abandons the exchange rate peg and devalues the currency, if its stock of foreign reserves falls below a critical level.[1] I assume domestic residents may do not know the true value

[1]A different assumption is that after the stock of foreign reserves reaches a critical level, the central bank floats the exchange rate and controls the resulting nominal

of this critical level and may have heterogeneous beliefs about it.[2] This critical level measures how resilient the central bank is in defending the currency peg in face of speculative attacks. With heterogeneous beliefs, it is likely that given the current level of foreign reserves, some residents expect the peg to remain while others expect the peg to collapse. As market opinions diverge, loan transactions shall emerge voluntarily within the private sector. Intuitively, people, who expect the peg to collapse and thus expect the currency to be devalued, have incentives to sell domestic money short. They want to borrow domestic money and sell it off in foreign exchange markets. Even with a high nominal interest rate, due to an expected devaluation, they anticipate handsome profits from the arbitrage. Others, who expect the peg to remain, willingly take the counterpart position: lend with an interest rate perceived to be high in both nominal and real terms.

I study first the benchmark case where all domestic residents' beliefs about this critical level of defense are the same and coincide with the true value. I show that as long as the central bank is resilient enough in defending the currency, the fixed exchange rate can survive perpetually. Otherwise the pegged exchange rate either remains viable forever or is brought down by an economy wide self-fulfilling run on the central bank at some arbitrary time.

And then I introduce heterogeneous beliefs. I assume that domestic residents' beliefs about the critical level of defense are uniformly distributed over an interval,

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money supply, as in Krugman (1979), Flood and Garber (1984), and Obstfeld (1986). Under this assumption, the floating exchange rate is an endogenous variable, and there can be no discrete devaluations after the fixed exchange rate collapses.

[2]One empirical evidence for heterogeneous expectations of the credibility of the currency peg is presented in Valev and Carlson (2008). It gives a series of survey data from 2001 to 2004 taken in Bulgaria which introduced a currency board in 1997, showing respondents disagree over the likelihood that the currency board would collapse in different horizons.

centered on the true value. I find that loan transactions within the private sector arising from heterogeneous beliefs make an exchange rate peg vulnerable to speculative attacks. In particular, I show that given a distribution of beliefs across domestic residents, a fixed exchange rate which remains viable forever if private loans are not allowed, may be brought down by self-fulfilling speculative attacks at some arbitrary time if private loans are allowed. And I also show that a peg which remains viable forever if private loans are allowed, must remain viable if private loans are not allowed.

The core reason is that the possibility to engage in loans pushes up the opportunity cost of holding domestic money—those who expect the peg to collapse perceive profits of selling domestic money short while those who expect the peg to remain enjoy high interest rates from the private loans—and thus lowers the aggregate demand for domestic money.

Interestingly, the heterogeneity of beliefs per se brings stability to the system of a fixed exchange rate. In particular, I show that a pegged exchange rate which is subject to self-fulfilling crises if domestic residents' beliefs are the same, may remain viable forever if a perturbation of beliefs is introduced. And I also show that a peg which remains viable forever if domestic residents' beliefs are the same, must remain viable if the beliefs are heterogeneous. Intuitively, differences in opinion make the public's moves less synchronous. A decline in foreign reserves might gather only a handful of domestic residents into the crowd to attack the central bank, since some people do not expect the peg to collapse given their own beliefs.

Hence what gives rise to the vulnerability of the system of a fixed exchange rate is not the heterogeneity of beliefs per se but the private transactions arising from it. The assumption of heterogeneous beliefs thus has two consequences: one is desynchronizing actions taken by the public while the other is creating incentives for side bets among private investors. The former brings stability while the latter generates destablizing private transactions. I show that provided the beliefs are not too diverse, the latter effect dominates. In particular, I show that a pegged exchange rate which is viable if domestic residents' beliefs are the same, is subject to self-fulfilling crises if the beliefs are heterogeneous and private loan transactions are allowed, provided the perturbation of beliefs is small enough.

In the market for loans, I choose collateral to enforce loan repayments, i.e., when loans are initiated, borrowers need to pledge certain properties as collateral to secure loan repayments; failures to repay give lenders the right to seize the pledged properties. Furthermore, I make collateral negotiable and thus endogenously determined, following Geanakoplos (1997, 2003, 2010).

From the modeling perspective, it is important to be explicit on enforcement mechanism when participants in asset markets have different beliefs and meanwhile unlimited short-selling is not excluded for exogenous reasons. Hart (1974) shows when short-selling is unlimited, the equilibrium might not exist if people have too much disagreement over the security returns while the equilibrium does exist if people's beliefs are identical. As Milne (1980) argues, the key assumption that gives rise to the non-existence result in Hart (1974) is that lenders never question borrowers' ability of paying off the loan. Milne (1980) argues even though no restriction on short-selling is exogenously assumed, some constraints on borrowing shall arise from lenders's perceptions of default risk. And Milne (1980) shows after introducing some enforcement mechanism an equilibrium does exist in an asset economy. One alternative mechanism considered by Milne (1980) is to assume lenders are able to access the information of borrowers' portfolios and a loan will be made only when the borrower is solvent in every contingency which the lender perceives will occur with a positive probability. The prime advantage of collateral being the enforcement mechanism is anonymity. As long as the collateral pledged is sound, there is no need for a lender to know the identity of the borrower, to have any information of the borrower's portfolios, and to believe the borrower is honest.

B. Credit default swaps

Credit default swaps, a class of financial derivatives, had attracted enormous attention from the public and policy-makers[3] since this recent financial crisis.[4]

A credit default swap is often described as a form of insurance which protects a lender if a borrower defaults.[5] For example, suppose an investor holds a bond issued by the General Motors Company (GM) and he worries that GM might file a bankruptcy soon. To hedge the default risk, the investor goes to buy a credit default swap on GM's bond. He makes periodic payments to the seller of the credit default swap in exchange for certain payoffs if GM defaults. In contrast to usual insurance contracts, credit default swaps have a peculiar feature. The buyer of a credit default swap is not required to have the insurable interest. It is unnecessarily true that the buyer suffers a loss from the default. In the example above, without being exposed to GM's default, an investor can buy a credit default swap on GM's bond merely because he speculates that GM is going to default. For this reason, a credit default swap can either be used as insurance, hedging default risk, or be used to gamble, betting against a security, (e.g. GM's bond in the example above).

The recent financial crisis was preceded by busts in housing markets and subprime

[3]See Che and Sethi (2011) for a short review of arguments for and against policies regulating credit default swaps.

[4]On September 16th 2008, American International Group, Inc. (AIG), the insurance giant, received bailout money up to $85 billion from the Federal Reserve Bank of New York, following the downgrade of its credit rating. The liquidity problem facing AIG at the time was to deposit additional collateral with its trading parties, mainly with those who had bought credit default swaps from AIG.

[5]See Stulz (2010) for a review of the mechanics of credit default swaps.

mortgages. And the credit default swaps involved in the crisis were those on mortgage-backed securities. The linkage between the decline in home prices and the trading of credit default swaps on mortgage-backed securities stirred a vast interest from the public. Nevertheless, theoretical works aimed at understanding this linkage are rare. One notable contribution is by Geanakoplos (2010). In a model of risk-neutral agents with heterogeneous beliefs about the return rate of a risky asset, Geanakoplos (2010) shows that when loans collateralized by the risky asset are allowed, the introduction of credit default swaps on these collateralized loans depresses the equilibrium price of the risky asset and eliminates transactions of collateralized loans in equilibrium.

In this work, I study credit default swaps in a general equilibrium model similar to Geanakoplos (2010). But unlike Geanakoplos (2010), I start with a general framework which allows for risk-averse agents and allows for more than two states of nature. The model features a market for a risky asset, (house), a market for loans collateralized by the risky asset, (mortgages), and a market for credit default swaps which reference the collateralized loans. Like Geanakoplos (2010), I only consider loans denominated in a risk-free asset, (cash), and exclude short sales against the risky asset. Following Geanakoplos (1997, 2003, 2010), collateral requirements, along with interest rates and loan quantities, are endogenously determined in the market for loans.

Individuals are assumed to be identical in all aspects except for the belief about the return rate of the risky asset. Due to heterogeneous beliefs, individuals have incentives to borrow and lend among themselves. Individuals with bullish views want to purchase the risky asset through borrowing and meanwhile use the obtained risky asset as collateral, i.e., buying the risky asset on margin. Meanwhile, individuals with bearish views want to lend, provided the interest rate is high enough and the collateral is sound enough. Also due to heterogeneous beliefs, individuals have incentives to buy and sell credit default swaps on these loans among themselves. Since these loans are collateralized by the risky asset, for bulls these loans have good chances to be repaid while for bears these loans are likely to go bad. Hence the former are willing to insure the repayments of these loans with the latter.

I make two assumptions about trading credit default swaps. First I assume that a sufficient amount of collateral must be posted to back up the promise made in a credit default swap so that the seller of the credit default swap would be able to deliver payoffs in all contingencies. Second, I assume that sellers of credit default swaps can economize on collateral, meaning that the remnant of the collateral posted for a credit default swap on a loan can be used as an equivalent substitute for the risky asset as collateral for the same loan. The consequences of these two assumptions are (1) buying credit default swaps to alleviate potential losses from loan defaults is equivalent with lending less and holding additional amounts of the risk-free asset,[6] and (2) selling credit default swaps is equivalent with lending under the corresponding loans.[7]

Due to these two facts, I show that the introduction of credit default swaps as insurance only has no effect on the equilibrium price of the risky asset and the market for collateralized loans. Note this result does not depend on how individuals' beliefs about the return rate of the risky asset are specified.

To analyze the effect of introducing credit default swaps both used to hedge default risk and used to bet against loans collateralized by the risky asset, I make a

[6]As it is noted in the book by Lewis (2010), the best way to avoid the risk of GM's default is not to lend to GM in the first place.

[7]In practice, credit default swaps are used to structure synthetic collateralized loans, an example of which is ABACUS 2007-AC1 in the center of the lawsuit against the Goldman Sachs Group, Inc. (Goldman Sachs) filed by the Securities and Exchange Commission (SEC). According to SEC, investors who bought ABACUS 2007-AC1 were in essence the sellers of the credit default swaps which referenced a variety of subprime mortgage-backed securities. The official document of the lawsuit is available at http://www.sec.gov.

concrete specification of individuals' beliefs about the return rate of the risky asset. I assume the economy is divided into two groups: "optimists" and "pessimists". Individuals within a group have the same point estimate about the return rate of the risky asset. Optimists, as their name suggests, have a higher estimate than pessimists do.

I solve analytically for the price of the risky asset in equilibrium when credit default swaps both are prohibited and are allowed. I find that as the market for credit default swaps is introduced, the price of the risky asset in general falls, but this is not always the case. When the population of pessimists is large enough, opening up the market for credit default swaps does nothing to the price of the risky asset and the market for loans collateralized by the risky asset. Interestingly, this is not because pessimists have no incentives to buy the credit default swaps but because optimists perceive that the return rate of buying the risky asset on margin dominates that of selling the credit default swaps and hence there is a lack of incentive on the side of sellers.

Further, unlike Geanakoplos (2010), I find that in general the introduction of credit default swaps does not eliminate transactions of collateralized loans in equilibrium. It is true only when the population of pessimists is small enough.

CHAPTER II

SPECULATIVE CURRENCY CRISES

This chapter is devoted to my study on speculative currency crises. The theme of this study is to theoretically analyze short-term collateralized loans among domestic residents arising from heterogeneous beliefs in currency crises.

A. The basic model

1. Setup

Consider a small open economy inhabited by a continuum of infinitely lived residents of mass 1. The time is discrete, meaning markets open only at a set of dates separated by a unit length of time, i.e., $t-1, t, t+1, ...$

The domestic central bank stands ready to buy and sell foreign exchanges to peg the spot nominal exchange rate, defined as the price of foreign currency (dollar) in terms of domestic currency (peso).

The economy produces and consumes a single trade-able good which perishes in a unit length of time. Assume purchasing power parity prevails at all times, and the dollar price of the good is normalized to 1. Hence the domestic price level is equal to the nominal exchange rate.

A foreign consol-type bond is available to domestic residents, which pays a interest rate with certainty in dollars in perpetuity. Precisely, each unit of the foreign bond gives i^* dollars as the interest in each date. The foreign bond is supplied elastically to the residents in the small economy at a given price, which is exogenously determined in the rest of the world. Normalize the price of the foreign bond to 1 dollar per unit.

I assume foreigners do not desire peso and peso-denominated assets. I assume that the local government does not issue bonds and there are no other domestic assets except the domestic currency and private loans among residents. Holding domestic money provides liquidity and thus is desired, even though it yields no interest. When domestic residents are homogeneous, in equilibrium there are no actual loan transactions, and thus it is safe to drop private loans from their portfolio decision. Below, I proceed to discuss the problem facing a typical domestic resident with private loans, while I discuss in detail how domestic residents trade loans later as I introduce heterogeneous beliefs.

A typical domestic resident lives forever and maximizes the lifetime utility:

$$\sum_{1,2,\ldots,\infty} \frac{u(z_t, m_t)}{(1+i^*)^{t-1}} \ , \tag{2.1}$$

where z denotes consumption and m denotes real cash balance defined as $\frac{M}{S}$, M denotes the holding domestic money and S denotes the nominal exchange rate. Above, I assume that holding domestic money gives utility directly. Further, I assume that the utility function is separable in z and m and that the intertemporal substitution of consumption is perfectly elastic, i.e.,

$$u(z, m) = z + l(m) \ . \tag{2.2}$$

A typical domestic resident is facing two budget constraints at each date: a stock constraint and a flow one. Let w denotes the wealth in real terms held by a domestic resident, which is equal to the sum of the real money stock and the foreign bond, i.e., for all t,

$$w_t = m_t + f_t \ , \tag{2.3}$$

where f denotes the holding of the foreign bond. Initially a typical domestic resident starts with some nominal money stock $M_0 > 0$ and some foreign bond $f_0 > 0$. I use w_0 to denote the initial wealth for a typical resident. Hence $w_0 = f_0 + \frac{M_0}{S_0}$. The flow constraint at date t gives,

$$w_t - w_{t-1} + z_t = i^* \cdot f_{t-1} - m_{t-1}\pi_t + y \ , \tag{2.4}$$

where y denotes the exogenous constant income flow received by a typical individual, π_t denotes the rate of depreciation of domestic money defined as $\pi_t \equiv \frac{S_t - S_{t-1}}{S_t}$.

At each date, a typical domestic resident chooses consumption and portfolio given the wealth that he is left with from the past. The necessary conditions for maximizing (2.1) subject to (2.3) and (2.4) imply: first consumption z_t is indeterminate; second the demand for money is given by

$$l'(m_t) \ = \ \frac{i^* + \pi_{t+1}}{1 + i^*} \ . \tag{2.5}$$

Let $\mathcal{L}(\cdot) = [l'(\cdot)(1 + i^*)]^{-1}$. Hence

$$m_t = \mathcal{L}(i^* + \pi_{t+1}) \ . \tag{2.6}$$

The local fiscal authority does not issue bonds, and the local central bank holds the foreign bond as reserves to peg the nominal exchange rate at $\bar{S} > 0$. The consolidated government flow budget constraint at date t gives,

$$f_t^g - f_{t-1}^g + g_t = m_t - m_{t-1} + m_{t-1}\pi_t + i^* \cdot f_{t-1}^g \ , \tag{2.7}$$

where f^g denotes the foreign bond held by the central bank and g denotes the government spending. I assume the central bank maintains a sufficient amount of foreign reserves just enough to absorb the entire money supply at all dates. Initially $\frac{M_0}{S_0} = f_0^g$. Without loss of generality, let $f_0^g = 1$. As a result, the government spending is financed by the interest earned from the foreign bond plus revenues from the inflation tax. Since the central bank pegs the exchange rate at a constant level $\bar{S}$, the inflation tax is zero, and hence,

$$i^* \cdot f_t^g = g_t \ . \tag{2.8}$$

Combine flow budget constraints for the private sector and the government, (2.3), (2.4), and (2.7), I get the balance of payments equation,

$$f_t^a - f_{t-1}^a = y - z_t - g_t + i^* \cdot f_{t-1}^a \ , \tag{2.9}$$

where f^a denotes the aggregate holding of the foreign bond by the entire economy, i.e., $f^a = f + f^g$. The long-run stationary equilibrium requires that $f_t^a = f_{t-1}^a$. Hence, in the long-run equilibrium,

$$z_t = y + i^* \cdot f_{t-1} \ . \tag{2.10}$$

And the real cash balances for a typical resident in the long-run stationary equilibrium should be $\mathcal{L}(i^*)$. Without loss of generality, let $\mathcal{L}(i^*) = 1$.

2. Speculative currency crises

In the basic setup described above, the central bank never abandons the pegged exchange rate. From now on, I assume that the decision for the domestic central bank to abandon the peg is dependent on the amount of foreign reserves. At the beginning of each date just before markets open, the domestic central bank announces the decision: either it continues to peg the nominal exchange rate at $\bar{S}$ or it devalues the currency. I assume in the event of devaluation, the domestic central bank will peg the nominal exchange rate at a new level thereafter and the new peg is known to the public, denoted by $\hat{S} > \bar{S}$. Let

$$\bar{\pi} \equiv \frac{\hat{S} - \bar{S}}{\hat{S}} \ . \tag{2.11}$$

The central bank decides to abandon the peg at date t provided f^g_{t-1} falls below a critical level, denoted by $\bar{f}^g \in [0, 1]$, which captures the resilience of the system of a fixed exchange rate. The value of $\bar{f}^g$ is not necessarily known to the public. Moreover, in the succeeding section, I assume that domestic residents might have different beliefs about $\bar{f}^g$, which induces heterogeneous expectations regarding the prospective exchange rates. Before heading forward, I analyze the case where domestic residents all have the same belief about $\bar{f}^g$ which also coincides with the true value. $\theta \in [0, 1]$ denotes the true value of $\bar{f}^g$.

Suppose at date t, the central bank devalues the currency. Since the public anticipates no more devaluation thereafter, and then the economy reaches the long-run stationary equilibrium immediately. All variables stay constant starting from date t. The real cash balances is equal to $\mathcal{L}(i^*)$, the consumption is equal to $(i^* f_t + y)$, and f_t is given by

$$(1 + i^*) f_t + \mathcal{L}(i^*) = (1 + i^*) f_{t-1} + (1 - \bar{\pi}) m_{t-1} \ . \tag{2.12}$$

Suppose at date t, the central bank still pegs the nominal exchange rate at $\bar{S}$. The public understand that the expected devaluation rate should be dependent on f^g_t, i.e.,

$$\pi_{t+1} = 1\{f^g_t < \theta\} \cdot \bar{\pi} + 1\{f^g_t \geq \theta\} \cdot 0 \ . \tag{2.13}$$

Meanwhile the demand for money at date t by the public is given by $\mathcal{L}(i^* + \pi_{t+1})$.

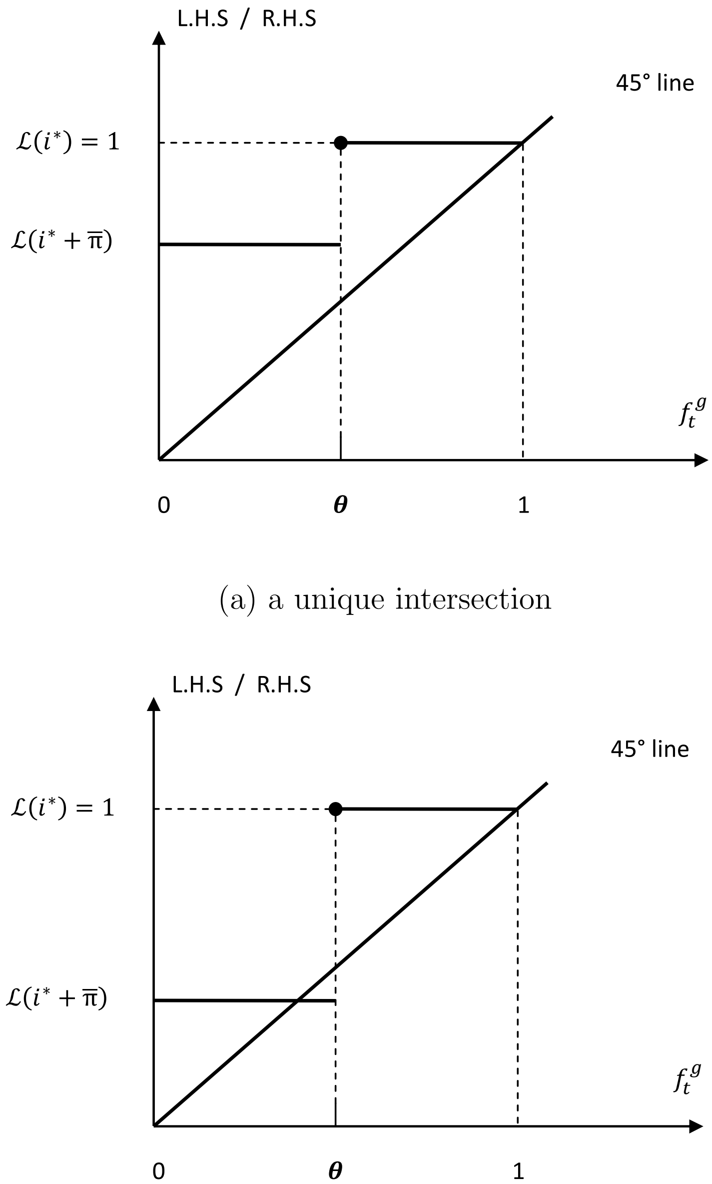
By the central bank's balance sheet, $m_t = f_t^g$. In equilibrium,

$$f_t^g = \mathcal{L}\big(\, i^* + \bar{\pi} \cdot 1\{f_t^g < \theta\}\,\big)\ . \tag{2.14}$$

For illustration, in Fig. 1, I graph the demand for money at date t, which is the right hand side of the equation above, against the stock of foreign reserves for the central bank at date t, i.e., f_t^g. The intersections of the curve with the 45° line are the possible solutions to the equation above and thus are the possible equilibria.

As Fig. 1 shows, depending on where the true value of $\bar{f}^g$ lies relative to $\mathcal{L}(i^*+\bar{\pi})$, there are in general two scenarios. If the system of the fixed exchange rate at $\bar{S}$ is resilient, i.e., θ is low, if all domestic residents expect the peg to collapse, the loss of foreign reserves is not sufficient to bring down the system, precisely, i.e., $\theta \le \mathcal{L}(i^*+\bar{\pi})$, and hence the central bank is not subject to self-fulfilling currency attacks. If the peg $\bar{S}$ is not resilient, i.e., $\theta > \mathcal{L}(i^* + \bar{\pi})$, the system of the fixed exchange rate can be brought down by an economy wide speculative attack at any arbitrary time, even if it might remain viable permanently in the absence of the attack. The results are formally stated in the following theorem.

Theorem 1. *When it it assumed that domestic residents believe $\bar{f}^g = \theta$, if $\theta \in (0, \mathcal{L}(i^* + \bar{\pi})]$, and then the equilibrium is unique: at all dates domestic residents all hold domestic money $\mathcal{L}(i^*)$ and the domestic central bank pegs the exchange rate at $\bar{S}$; if $\theta \in (\mathcal{L}(i^* + \bar{\pi}), 1]$, and then there are two types of equilibria: (1) at all dates domestic residents all hold domestic money $\mathcal{L}(i^*)$ and the domestic central bank pegs the exchange rate at $\bar{S}$, and (2) at some arbitrary date, domestic residents all lower the demand for domestic money to $\mathcal{L}(i^*+\bar{\pi})$, which triggers the domestic central bank to abandon the peg $\bar{S}$ and to devalue the currency at the rate $\bar{\pi}$ at the succeeding date.*



(b) two intersections

Fig. 1. Equilibria with homogeneous belief.

Above I obtain the standard result of multiple equilibria in the literature of currency crises. Below, I depart from the assumption that domestic residents' beliefs about $\bar{f}^g$ are the same.

B. Heterogeneous beliefs

From this section on, I assume that domestic residents' beliefs about $\bar{f}^g$ are uniformly distributed over $[\theta - \delta, \theta + \delta] \subset [0, 1]$. $K(\cdot)$ denotes the mass of domestic residents whose beliefs on $\bar{f}^g$ are less than a given value. For the sake of exposition, θ_x is used to denote the belief about $\bar{f}^g$ by domestic resident $x \in [0, 1]$.

1. Without private loans

To understand the role that private loans play in currency crises, in this subsection, I first study the case with heterogeneous beliefs but without private loans. Since domestic residents' beliefs on $\bar{f}^g$ may not coincide with the true value, it is possible that as some domestic residents run on the central bank, the loss of foreign reserves is not sufficiently large to trigger the collapse of the peg. After unsuccessful speculative attacks, domestic residents should refine or correct their beliefs about $\bar{f}^g$. Nevertheless, I restrict attention to the type of equilibria in which speculative currency attacks take place at most once.

Suppose at date t, the central bank still pegs the nominal exchange rate at $\bar{S}$. For domestic residents $\theta_x \leq f_t^g$, they hold real cash balances $\mathcal{L}(i^*)$, while for domestic residents $\theta_x > f_t^g$, $\mathcal{L}(i^* + \bar{\pi})$. Hence in the aggregate,

$$f_t^g = K(f_t^g) \cdot \mathcal{L}(i^*) + (1 - K(f_t^g)) \cdot \mathcal{L}(i^* + \bar{\pi}) \ , \tag{2.15}$$

where $K(f_t^g) = \frac{f_t^g - \theta + \delta}{2\delta}$ if $f_t^g \in [\theta - \delta, \theta + \delta]$; $K(f_t^g) = 0$ if $f_t^g < \theta - \delta$, and $K(f_t^g) = 1$ if $f_t^g > \theta + \delta$.

In Figs. 2-3, I graph the aggregate demand for money at date t, which is the right hand side of the equation above, against the stock of foreign reserves for the central bank at date t, i.e., f_t^g. The intersections of the curve with the 45° line are the possible solutions to (2.15) and thus are the possible equilibria. Fig. 2 shows the case with no speculative attacks, while Fig. 3 shows two other cases. For both cases, there exists a type of equilibria in which only a fraction of the economy expecting the central bank to abandon the peg $\bar{S}$. The difference is that, in this type of equilibria, in one case the central bank in fact abandons the peg while in the other case it does not. Theorem 2 states the possible equilibria, given a range of values for θ.

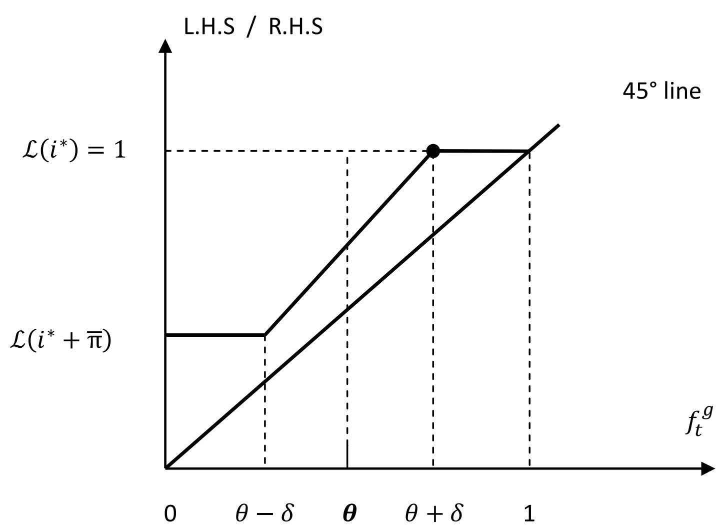
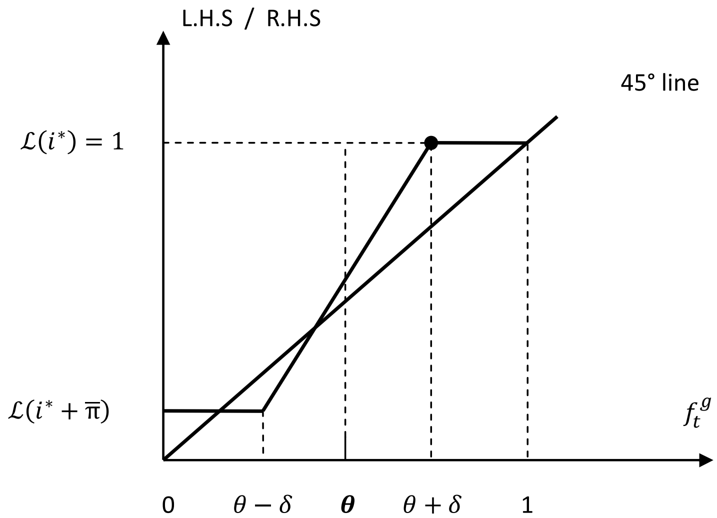
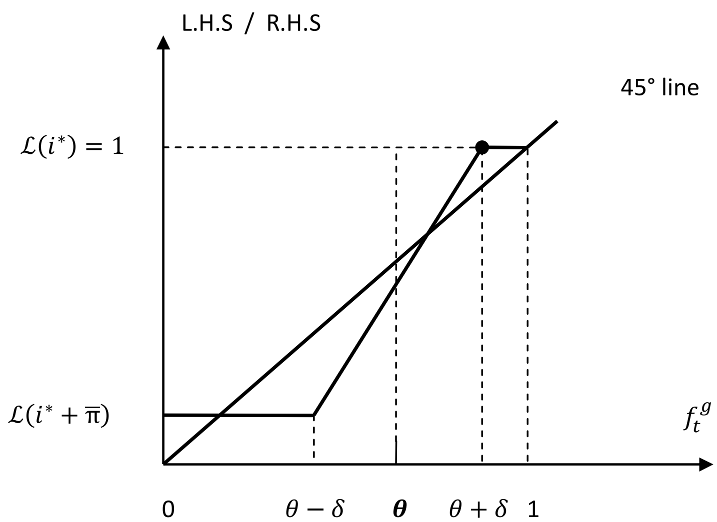


Fig. 2. Unique equilibrium with heterogeneous beliefs without private loans.



(a) successful speculative attacks



(b) one successful attack and the other unsuccessful

Fig. 3. Multiple equilibria with heterogeneous beliefs without private loans.

Theorem 2. *When it is assumed that domestic residents' beliefs about $\bar{f}^g$ are uniformly distributed over $[\theta - \delta, \theta + \delta] \subset [0, 1]$, if $\theta \in (0, \mathcal{L}(i^* + \bar{\pi}) + \delta)$, the equilibrium is unique: at all dates domestic residents all hold domestic money $\mathcal{L}(i^*)$ and the do-*

mestic central bank pegs the exchange rate at $\bar{S}$*; if* $\theta \in [\mathcal{L}(i^* + \bar{\pi}) + \delta, 1)$*, there are three types of equilibria: (1) at all dates domestic residents all hold domestic money* $\mathcal{L}(i^*)$ *and the domestic central bank pegs the exchange rate at* $\bar{S}$*, (2) at some arbitrary date, domestic residents all lower the demand for domestic money to* $\mathcal{L}(i^* + \bar{\pi})$*, which triggers the domestic central bank to abandon the peg* $\bar{S}$ *and to devalue the currency at the rate* $\bar{\pi}$ *at the succeeding date, and (3) at some arbitrary date, domestic residents of mass* $\frac{\theta - \delta - \mathcal{L}(i^* + \bar{\pi})}{\mathcal{L}(i^*) - \mathcal{L}(i^* + \bar{\pi}) - 2\delta}$ *hold domestic money* $\mathcal{L}(i^*)$ *while the rest lower the demand for domestic money to* $\mathcal{L}(i^* + \bar{\pi})$*, which in the aggregate depletes the foreign reserves to*

$$\frac{\mathcal{L}(i^*)(\theta - \delta) - \mathcal{L}(i^* + \bar{\pi})(\theta + \delta)}{\mathcal{L}(i^*) - \mathcal{L}(i^* + \bar{\pi}) - 2\delta} \,, \tag{2.16}$$

which triggers the domestic central bank to abandon the peg $\bar{S}$ *and to devalue the currency at the rate* $\bar{\pi}$*, provided* $\theta < \frac{\mathcal{L}(i^* + \bar{\pi}) + \mathcal{L}(i^*)}{2}$*.*

One main implication of Theorem 2 is as follows. If θ, the index of resilience of the system, lies in the interval $(\mathcal{L}(i^* + \bar{\pi}), \mathcal{L}(i^* + \bar{\pi}) + \delta)$, the system of the fixed exchange rate at $\bar{S}$ is subject to self-fulfilling currency crises when domestic residents' beliefs are homogeneous and coincide with θ, while the peg $\bar{S}$ shall remain viable permanently when a perturbation of beliefs is introduced. Intuitively, this means that a heterogeneity of beliefs per se would make self-fulfilling currency crises less likely to take place. Note as δ approaches to zero, the interval $(\mathcal{L}(i^* + \bar{\pi}), \mathcal{L}(i^* + \bar{\pi}) + \delta)$ vanishes to a void set.

It is interesting to note that when $\theta \geq \mathcal{L}(i^* + \bar{\pi}) + \delta$, as θ decreases, for the type of equilibria in which only a fraction of the economy expecting the central bank to abandon the peg $\bar{S}$, the speculative attack goes from failure to success. This appears to say counter-intuitively that as the system of the fixed exchange rate at $\bar{S}$ becomes more resilient, the system becomes more likely to collapse. The key is to note that as θ decreases, not only the system in fact becomes more resilient, but also the public anticipate that the losses of reserves must be larger to bring the system down. Hence if there is a speculative attack launched by a fraction of domestic residents, the stock of foreign reserves must be lower and thus the fraction must be larger as θ decreases. That the fraction of speculators gets larger implies that the decline in foreign reserves must exceed that in θ. This explains why as θ decreases the speculative attack goes from failure to success.

2. Market for private loans

All private loans mature in a unit length of time, i.e., a loan initiated at date t matures at date $t+1$. Private loans are denominated in pesos.[1] If a domestic resident lends at date t, he pays some pesos at date t and is supposed to receive some pesos as the repayment at date $t+1$; if the domestic resident borrows, he instead receives some pesos and is required to hold some assets as the collateral at date t, and he is supposed to repay some pesos at date $t+1$. When the loan is not repaid at date $t+1$, the collateral is seized to pay off the loan. I assume peso and the foreign bond are enforce-able collateral for loans among domestic residents.

A peso-denominated private loan is defined by a triple, (R, c_f, c_m), which states the repayment rate and the characteristics of collateral. Table I gives the definition for each contract terms. Geanakoplos (2010) points out that the key to endogenize

[1] Dollar-denominated loans are redundant, provided I assume peso-denominated loans can not be used as collateral. If a domestic resident wants to borrow in dollars, he needs to pay a gross interest rate, not smaller than $(1+i^*)$. To back up the loan repayments, he can post either the foreign bond or peso as the collateral. But the reason why some domestic residents want to borrow in dollars presumably is to take the advantage of the interest rate gap between the foreign bond and some domestic peso-denominated loans. Since they are unable to buy peso-denominated loans on margin, these domestic residents do not find incentives to borrow in dollars any more.

Table I. Three contract terms in a loan

Variable	
R	Pesos to repay / Notional value
c_f	Peso value of the foreign bond posted as collateral / Notional value
c_m	Pesos posted as collateral / Notional value

collateral is to index promises by their collateral as we index commodities by their qualities. Barro (1976) makes a similar point that a price of a loan should not only include the interest rate but also the characteristics of collateral.

For example, suppose the notional value of a loan $(1.2, 0.8, 0.1)$ is 100. $R = 1.2$ means 120 pesos needs to be repaid when the loan is due, $c_m = 0.1$ implies 10 pesos is put up as the collateral when the loan is initiated. $c_f = 0.8$ means the amount of the foreign bond pledged as the collateral is worth 80 pesos at time when the loan is initiated.

Domestic residents borrow and lend not in a bilateral way but in Walrasian markets. For each loan, given its associated price, domestic residents submit their orders of borrowing or lending to a Walrasian auctioneer. For example, if domestic residents want to borrow or to lend under loan $(1.2, 0.8, 0.1)$, they need to tell the auctioneer how much they want to borrow or to lend. The notional value is the number that domestic residents use to communicate with the auctioneer about the quantity to borrow or to lend. If a borrowing order is placed under loan $(1.2, 0.8, 0.1)$ of a notional value 100, the auctioneer writes -100 in its calculation of market clearing. If a lending order is placed of a notional value 150, the auctioneer adds $+150$. The market for loan $(1.2, 0.8, 0.1)$ is cleared, when after summing up the numbers from all orders the auctioneer gets zero.

The price of a loan is pesos per notional value to pay for those who want to lend (or to receive for those who want to borrow) when the loan is initiated. For example, if the price of loan $(1.2, 0.8, 0.1)$ is 0.5 pesos per notional value, and if a domestic resident wants to lend under loan $(1.2, 0.8, 0.1)$ of a notional value 100, he needs to pay 50 pesos. Formally, there is a mapping which associates a loan with a price,

$$p_t(R, c_f, c_m) : \Re^3_+ \to \Re_+ \ , \tag{2.17}$$

where $p_t(R, c_f, c_m)$ is the price for loan (R, c_f, c_m) at date t. The auctioneer announces the price of a loan and adjusts it until the market for the loan is cleared. After a market clearing price is reached, the auctioneer collects payments from those who place a lending order and distribute them to those who place a borrowing order when the loan is initiated and collects repayments from borrowers and distribute them to lenders when the loan matures. The promised repayment of a loan does not necessarily coincide with the actual repayment. Under the collateral mechanism, borrowers will repay the loan only when the collateral is worth more than the amount due, i.e., whenever

$$R \leq c_f \cdot \frac{S_{t+1}}{S_t} \cdot (1 + i^*) + c_m \ , \tag{2.18}$$

the loan will be paid off, otherwise borrowers default and the collateral is seized. Note whether or not the loan is paid off is contingent on $\frac{S_{t+1}}{S_t}$.

$r(\pi_{t+1}; R, c_f, c_m)$ denotes the actual return rate for loan (R, c_f, c_m), and

$$r(\pi_{t+1}; R, c_f, c_m) = \min\{R, \frac{1 + i^*}{1 - \pi_{t+1}} \cdot c_f + c_m\} \ . \tag{2.19}$$

Note $r(\pi_{t+1}; R, c_f, c_m)$ is homogeneous of degree one with respect to (R, c_f, c_m). So is $p_t(R, c_f, c_m)$, if certain types of arbitrage activities are allowed. Without loss of generality, I assume domestic residents restrict their choices to the set of loans the price of which is 1. Ω_t denotes this set at date t,

$$\Omega_t = \{(R, c_f, c_m) \in \Re^3_+ \parallel p_t(R, c_f, c_m) = 1\} \ . \tag{2.20}$$

3. Individual problem with private loans

Since domestic residents are identical in all aspects except for beliefs about $\bar{f}^g$, the analysis below is applicable to all.

Given Ω_t, the set of private loans available in the market at date t, a typical domestic resident decides the quantity of borrowing or lending for each peso-denominated loan. The choices are represented by a mapping,

$$B_t\big(R, c_f, c_m\big) : \Omega_t \longrightarrow \Re \ , \tag{2.21}$$

which associates a loan with the notional value of the loan to borrow or to lend. If $B_t\big(R, c_f, c_m\big) < 0$, it indicates borrowing. I define

$$b_t(R, c_f, c_m) = \frac{B_t(R, c_f, c_m)}{S_t} \ , \tag{2.22}$$

where $b_t(R, c_f, c_m)$ is the holding of private loan (R, c_f, c_m) in real terms.

The stock budget constraint becomes,

$$w_t = f_t + m_t + \sum_{\Omega_t} b_t(R, c_f, c_m) \ . \tag{2.23}$$

The flow budget constraint becomes,

$$\begin{aligned} & f_t + m_t + \sum_{\Omega_t} b_t(R, c_f, c_m) + z_t = \\ & (1 + i^*) f_{t-1} + (1 - \pi_t) m_{t-1} + y + \sum_{\Omega_{t-1}} r(\pi_t; R, c_f, c_m) \cdot (1 - \pi_t) \cdot b_{t-1}(R, c_f, c_m) \ . \end{aligned} \tag{2.24}$$

In addition, there are two collateral constraints,

$$m_t \geq -\sum_{\Omega_t} \left[c_m \cdot \min\left\{ b_t(R, c_f, c_m), 0 \right\} \right] , \tag{2.25}$$

$$f_t \geq -\sum_{\Omega_t} \left[c_f \cdot \min\left\{ b_t(R, c_f, c_m), 0 \right\} \right] . \tag{2.26}$$

The following lemma gives the Euler-Lagrange equations. Let λ denote the Lagrange multiplier with respect to the flow constraint (2.24), μ^m the Lagrange multiplier with respect to collateral constraint (2.25), and μ^f the Lagrange multiplier with respect to collateral constraint (2.26).

Lemma 1. *If* $\{ z_t, m_t, f_t, b_t(\cdot), \lambda_t, \mu_t^f, \mu_t^m \}_1^\infty$ *maximizes (2.1) subject to (2.24), (2.25), and (2.26), it should satisfy following conditions,*

$$1 - \lambda_t \leq 0 , \tag{2.27}$$

where the equality holds if $z_t > 0$*;*

$$\lambda_{t+1} - \lambda_t + \mu_t^f \leq 0 , \tag{2.28}$$

where the equality holds if $f_t > 0$*;*

$$\frac{1 - \pi_{t+1}}{1 + i^*} \lambda_{t+1} + l'(m_t) - \lambda_t + \mu_t^m \leq 0 , \tag{2.29}$$

where the equality holds if $m_t > 0$*; for any* $(R, c_f, c_m) \in \Omega_t$ *such that* $b_t(R, c_f, c_m) \neq 0$*, it must be,*

$$r\left(\pi_{t+1}; R, c_f, c_m\right) \frac{1 - \pi_{t+1}}{1 + i^*} \lambda_{t+1} - \lambda_t + (\mu_t^m c_m + \mu_t^f c_f) \cdot 1\{b_t(R, c_f, c_m) < 0\} = 0 ; \tag{2.30}$$

for any $(R, c_f, c_m) \in \Omega_t$ *such that* $b_t(R, c_f, c_m) = 0$*, it must be,*

$$r\big(\pi_{t+1}; R, c_f, c_m\big)\frac{1-\pi_{t+1}}{1+i^*}\lambda_{t+1} - \lambda_t \leq 0\ , \tag{2.31}$$

$$r\big(\pi_{t+1}; R, c_f, c_m\big)\frac{1-\pi_{t+1}}{1+i^*}\lambda_{t+1} - \lambda_t + \mu_t^m c_m + \mu_t^f c_f \geq 0\ . \tag{2.32}$$

C. Equilibria with private loans

As in the case where private loans are not allowed, I restrict attention to the type of equilibria in which speculative currency attacks take place at most once. Suppose up to date t, the central bank still pegs the exchange rate at $\bar{S}$. The following lemma proves the impossibility of no transaction of private loans whenever there exists a division among domestic residents at date t: some expect the peg $\bar{S}$ to remain at date $t+1$ while the rest expect it to collapse.

Lemma 2. *In equilibria with collateralized private loans, and* $f_t^g \in (\theta-\delta, \theta+\delta)$*, it is impossible that no private loan is traded at date* t*.*

Proof. Prove by contradiction. Suppose, in an equilibrium, $f_t^g \in (\theta-\delta, \theta+\delta)$, there is no transaction of private loans among domestic residents. Due to $f_t^g \in (\theta-\delta, \theta+\delta)$, $\lambda_{t+1} = 1$ since the run on the central bank would not take place after date t. No transaction of private loans implies that for all domestic residents $\mu_t^f = \mu_t^m = 0$ and $\lambda_t = 1$. Hence $\forall \theta_x \in [\theta-\delta, \theta+\delta]$, $\forall (R, c_f, c_m) \in \Omega_t$,$r(\pi_{t+1}; R, c_f, c_m) \cdot \frac{1-\pi_{t+1}}{1+i^*} = 1$. Note $\pi_{t+1} = \bar{\pi} \cdot 1\{f_t^g < \theta_x\} + 0 \cdot 1\{f_t^g \geq \theta_x\}$. Contradiction. □

The following two lemmas state the possible patterns of loan transaction in equilibrium. The proofs are in Appendix A. As $f_t^g \in (\theta-\delta, \theta+\delta)$, there is a line dividing the economy into two brigades: domestic residents $\theta_x \leq f_t^g$ expect $\pi_{t+1} = 0$ while domestic residents $\theta_x > f_t^g$ expect $\pi_{t+1} = \bar{\pi}$. In preview, the type of transactions of substance is that domestic residents $\theta_x > f_t^g$ borrow. Nevertheless, it is possible that

$\theta_x \leq f_t^g$ borrow, but the transactions can be omitted without impact and thus are redundant.

Lemma 3. *In the type of equilibria in which speculative currency attacks take place at most once and collateralized private loans are allowed to trade freely among domestic residents, if* $f_t^g \in (\theta - \delta, \theta + \delta)$ *and domestic residents* $\theta_x \leq f_t^g$ *borrow from domestic residents* $\theta_x > f_t^g$ *under some loan* $(R, c_f, c_m) \in \Omega_t$*, it must be that*

- *either* $R \geq \frac{1+i^*}{1-\bar{\pi}}$ *and* $c_f(1+i^*) + c_m(1-\bar{\pi}) = (1+i^*)$*,*
- *or* $R = \frac{1+i^*}{1-\bar{\pi}}$*,* $c_f = 0$*, and* $c_m(1-\bar{\pi}) > (1+i^*)$*.*

When $c_m > 0$*, for domestic residents* $\theta_x \leq f_t^g$*,* $\mu_t^m = 0$*,* $\lambda_t = \frac{1}{1-\bar{\pi}}$*, and domestic residents* $\theta_x \leq f_t^g$ *must lend to domestic residents* $\theta_x > f_t^g$ *under other loans.*

Lemma 3 states the loans that domestic residents $\theta_x \leq f_t^g$ would possibly borrow in equilibrium. According to Lemma 3, if they borrow, domestic residents $\theta_x \leq f_t^g$ lower the demand for money at date t from $\mathcal{L}(i^*)$ to $\mathcal{L}(\frac{i^*+\bar{\pi}}{1-\bar{\pi}})$. And they do not hold the foreign bond except as the collateral. It shall be seen later that the transactions in which domestic residents $\theta_x \leq f_t^g$ borrow are of no substance and can be omitted.

Lemma 4. *In the type of equilibria in which speculative currency attacks take place at most once and collateralized private loans are allowed to trade freely among domestic residents, if* $f_t^g \in (\theta - \delta, \theta + \delta)$ *and domestic residents* $\theta_x > f_t^g$ *borrow from domestic residents* $\theta_x \leq f_t^g$ *under some loan* $(R, c_f, c_m) \in \Omega_t$*, there can be two scenarios: (1) for domestic residents* $\theta_x > f_t^g$*,* $\mu_t^f = \mu_t^m = 0$*, either* $R = \frac{1+i^*}{1-\bar{\pi}}$ *and* $c_f(1+i^*) + c_m \geq \frac{1+i^*}{1-\bar{\pi}}$*, or* $c_f = 0$*,* $c_m = \frac{1+i^*}{1-\bar{\pi}}$*, and* $R > \frac{1+i^*}{1-\bar{\pi}}$*; (2)for domestic residents* $\theta_x > f_t^g$*,* $\mu_t^f = \mu_t^m(1+i^*) > 0$*, either* $R = c_f(1+i^*) + c_m < \frac{1+i^*}{1-\bar{\pi}}$*, or* $c_f = 0$*,* $R > c_m$*, and* $0 < c_m < \frac{1+i^*}{1-\bar{\pi}}$*.*

Combining Lemma 2-4, given a value $f_t^g \in (\theta - \delta, \theta + \delta)$, what happens in the market for private loans falls into two categories: (1) for domestic residents $\theta_x > f_t^g$, $\mu_t^f = \mu_t^m = 0$, i.e., when both collateral constraints are slack, and (2) for domestic residents $\theta_x > f_t^g$, $\mu_t^f, \mu_t^m > 0$, i.e., when both collateral constraints are binding. In the first case, the set of private loans available, Ω_t, should satisfy,

$$\Omega_t = \{ \, (R, c_f, c_m) \in \Re_+^3 \parallel r(\bar{\pi}; R, c_f, c_m) = \frac{1 + i^*}{1 - \bar{\pi}} \, \} \, , \tag{2.33}$$

and thus domestic residents $\theta_x > f_t^g$ view all private loans in Ω_t indifferent to the foreign bond. Fig. 4 depicts Ω_t in a 3-dimensional space. By Lemma 3, the transactions in which domestic residents $\theta_x \leq f_t^g$ borrow can only happen in the first case.

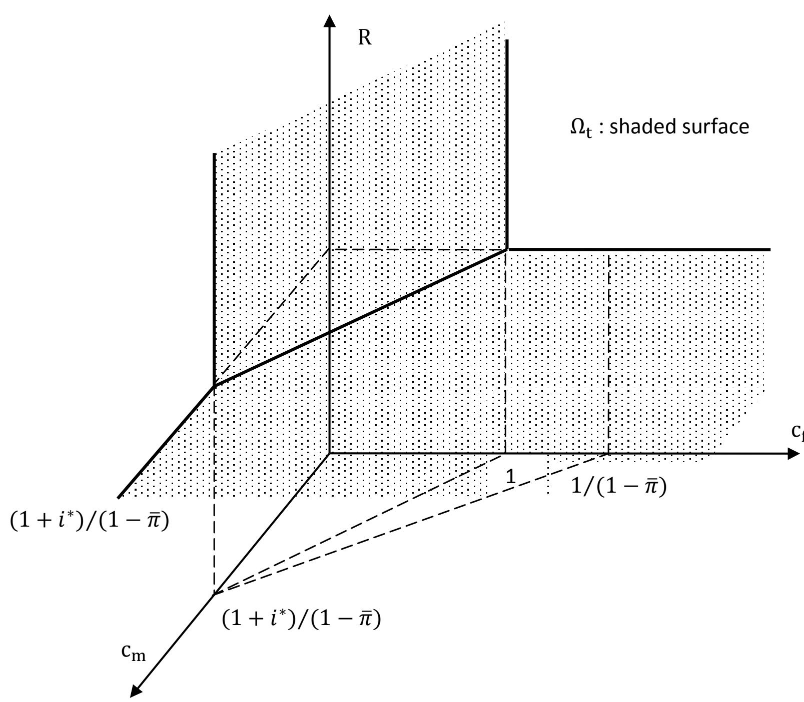


Fig. 4. Ω_t when for domestic residents $\theta_x > f_t^g$ both collateral constraints are slack.

Further, Ω_t can be classified into three parts, not necessarily exclusive, namely D_1, D_2, and D_3.

$$D_1 \equiv \{ (R, c_f, c_m) \in \Re^3_+ \| R \geq \frac{1+i^*}{1-\bar{\pi}}, c_f(1+i^*) + c_m(1-\bar{\pi}) = 1+i^* \}$$
$$\cup \{ (R, c_f, c_m) \in \Re^3_+ \| R = \frac{1+i^*}{1-\bar{\pi}}, c_f = 0, c_m > \frac{1+i^*}{1-\bar{\pi}} \} , \tag{2.34}$$
$$D_2 \equiv \{ (R, c_f, c_m) \in \Re^3_+ \| R = \frac{1+i^*}{1-\bar{\pi}}, c_f(1+i^*) + c_m < \frac{1+i^*}{1-\bar{\pi}} < c_f \} , \tag{2.35}$$
$$D_3 \equiv \{ (R, c_f, c_m) \in \Re^3_+ \| R = \frac{1+i^*}{1-\bar{\pi}}, c_f(1+i^*) + c_m \geq \frac{1+i^*}{1-\bar{\pi}} \}$$
$$\cup \{ (R, c_f, c_m) \in \Re^3_+ \| R > \frac{1+i^*}{1-\bar{\pi}}, c_f = 0, c_m = \frac{1+i^*}{1-\bar{\pi}} \} . \tag{2.36}$$

D_1 represents all loans under which domestic residents $\theta_x \leq f_t^g$ would possibly borrow in the first case. D_3 represents all loans under which domestic residents $\theta_x \leq f_t^g$ willingly lend to those $\theta_x > f_t^g$ in the first case.

Any loans in D_3 are preferred by domestic residents $\theta_x \leq f_t^g$ over the foreign bond and hence the incentives for domestic residents $\theta_x \leq f_t^g$ to borrow do not come from financing the purchases of the foreign bond but come either from financing the purchases of the private loans in D_3 or from the liquidity concerns. That private loans can not be used as collateral and that for loans in Ω_t, $cf + c_m \geq 1$, imply that there is no funds left to purchase the private loans in D_3 from the revenue of borrowing. Therefore, domestic residents $\theta_x \leq f_t^g$ borrow only because they invest too much in the private loans in D_3, which results in a shortage of cash. Hence the quantities of borrowing for domestic residents $\theta_x \leq f_t^g$ are residuals after the quantities of lending under the loans in D_3 and the holding of domestic money are determined. As a result, it is safe to set the quantities of borrowing for domestic residents $\theta_x \leq f_t^g$ to zero.

For domestic residents $\theta_x \leq f_t^g$, the loans in D_3 are the same and they are willing to use all resources available at date t, subtracting the demand for domestic money, to lend under the loans in D_3. Meanwhile, domestic residents $\theta_x > f_t^g$ must hold a sufficient amount of collateral to absorb the borrowing orders under the loans in D_3, which leads to a condition,

$$\frac{1-\bar{\pi}}{\bar{\pi}} \cdot \frac{(1+i^*)f_0 + m_0 + y - \frac{i^*}{1+i^*}\mathcal{L}(i^* + \bar{\pi})}{(1+i^*)f_0 + m_0 + y - \mathcal{L}(\frac{i^*+\bar{\pi}}{1-\bar{\pi}})} \geq \frac{K(f_t^g)}{1-K(f_t^g)} \ . \tag{2.37}$$

For the convenience of exposition, let

$$\beta \equiv \frac{(1-\bar{\pi}) \cdot [(1+i^*)f_0 + m_0 + y - \frac{i^*}{1+i^*}\mathcal{L}(i^* + \bar{\pi})]}{(1+i^*)f_0 + m_0 + y - \bar{\pi} \cdot \mathcal{L}(\frac{i^*+\bar{\pi}}{1-\bar{\pi}}) - (1-\bar{\pi})\frac{i^*}{1+i^*}\mathcal{L}(i^* + \bar{\pi})} \ . \tag{2.38}$$

According to Lemma 4, in the second case where for domestic residents $\theta_x > f_t^g$, $\mu_t^f, \mu_t^m > 0$, for a given value of $f_t^g \in (\theta - \delta, \theta + \delta)$, there exists a constant, $\hat{R} \in (1+i^*, \frac{1+i^*}{1-\bar{\pi}})$ such that domestic residents $\theta_x > f_t^g$ borrow from those $\theta_x \leq f_t^g$ under the loans in D_4, defined as follows,

$$\begin{aligned} D_4 \equiv & \{ \ (R, c_f, c_m) \in \Re_+^3 \| R = c_f(1+i^*) + c_m = \hat{R} \ \} \\ & \cup \{ \ (R, c_f, c_m) \in \Re_+^3 \| R > \hat{R}, c_f = 0, c_m = \hat{R} \ \} \ . \end{aligned} \tag{2.39}$$

Moreover, in the second case, for domestic residents $\theta_x > f_t^g$, the demand for money is equal to $\mathcal{L}(\frac{(1+i^*)(\hat{R}-1)\bar{\pi}}{\hat{R}-(1+i^*)})$ while for domestic residents $\theta_x \leq f_t^g$, the demand for money is equal to $\mathcal{L}(\hat{R}-1)$. Further, the clearing condition in the market for loans gives,

$$\frac{\hat{R}-(1+i^*)}{1+i^*} = \frac{1-K(f_t^g)}{K(f_t^g)} \cdot \frac{(1+i^*)f_0 + m_0 + y - \frac{i^*}{1+i^*}\mathcal{L}(\frac{(1+i^*)(\hat{R}-1)\bar{\pi}}{\hat{R}-(1+i^*)})}{(1+i^*)f_0 + m_0 + y - \mathcal{L}(\hat{R}-1)} \ . \tag{2.40}$$

The equation above defines a mapping from $f_t^g \in (\theta - \delta, \theta + \delta)$ to $\hat{R}$. Let $s(\cdot)$ denote this mapping, i.e., $\hat{R} = s(f_t^g)$. In the second case, the set of loans available in the market can be as follows. Fig. 5 depicts Ω_t in the second case.

$$\Omega_t = \{\ (R, c_f, c_m) \in \Re^3_+ \| R = \hat{R}, c_f(1+i^*) + c_m \geq \hat{R}\ \} \cup$$
$$\{\ (R, c_f, c_m) \Re^3_+ \| R > \hat{R}, R\tfrac{1-\bar{\pi}}{1+i^*} + \tfrac{(1+i^*)-\hat{R}(1-\bar{\pi})}{\hat{R}-(1+i^*)}(c_f + \tfrac{c_m}{1+i^*}) = \tfrac{\hat{R}\bar{\pi}}{\hat{R}-(1+i^*)}, c_f\hat{R} + c_m \geq \hat{R}\ \}$$
$$\cup\ \{\ (R, c_f, c_m) \Re^3_+ \| R\tfrac{1-\bar{\pi}}{1+i^*} + \tfrac{(1+i^*)-\hat{R}(1-\bar{\pi})}{\hat{R}-(1+i^*)}(c_f + \tfrac{c_m}{1+i^*}) > \tfrac{\hat{R}\bar{\pi}}{\hat{R}-(1+i^*)}, c_f\hat{R} + c_m = \hat{R}\ \}\ . \tag{2.41}$$

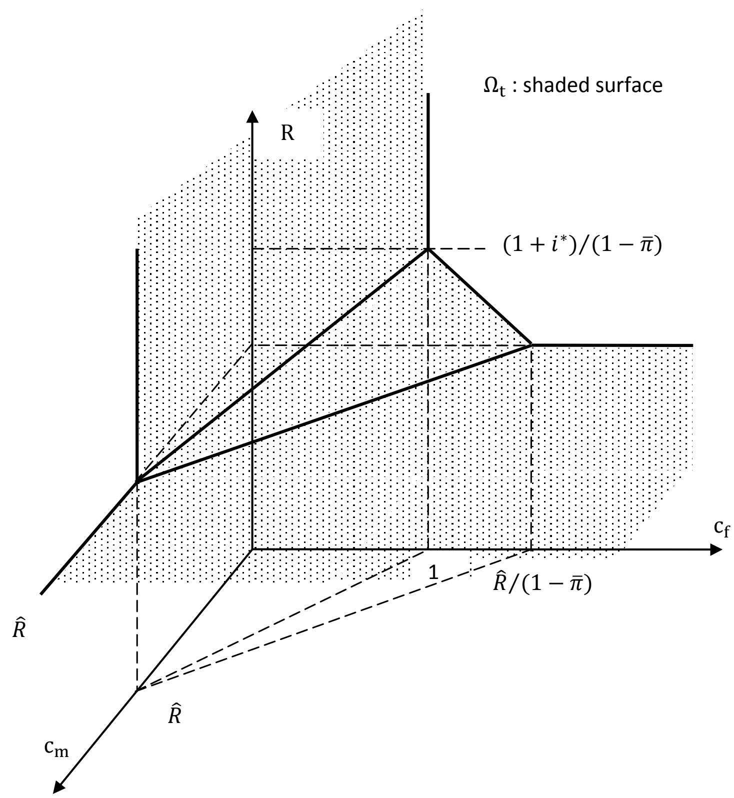


Fig. 5. Ω_t when for domestic residents $\theta_x > f^g_t$ both collateral constraints are binding.

The following lemma states provided $K(f^g_t) \leq \beta$, what happens in the market for private loans falls into the first category, otherwise the second one.

Lemma 5. *In the type of equilibria in which speculative currency attack takes place at most once and collateralized private loans are allowed to trade freely among domestic residents, if* $f_t^g \in (\theta - \delta, \theta + \delta)$*, there are only two possibilities.*

1. *When* $0 < K(f_t^g) \leq \beta$*: for domestic residents* $\theta_x \leq f_t^g$*, the holding of domestic money is* $\mathcal{L}(\frac{i^*+\pi}{1-\pi})$*; for domestic residents* $\theta_x > f_t^g$*, the holding domestic money is* $\mathcal{L}(i^* + \pi)$*.*

2. *When* $\beta < K(f_t^g) < 1$*: for domestic residents* $\theta_x \leq f_t^g$*, the holding of domestic money is* $\mathcal{L}(\hat{R} - 1)$*; for domestic residents* $\theta_x > f_t^g$*, the holding domestic money is* $\mathcal{L}(\frac{(1+i^*)(\hat{R}-1)\bar{\pi}}{\hat{R}-(1+i^*)})$*, where* $\hat{R}$ *is defined in (2.40).*

Note β *is defined in (2.38).*

Immediately from Lemma 5, the aggregate demand for money at date t can be written as a function of f_t^g.

$$
\begin{aligned}
& m_t(f_t^g; \delta, \bar{\pi}) = 1\{K(f_t^g) = 0\} \cdot \mathcal{L}(i^* + \bar{\pi}) + 1\{K(f_t^g) = 1\} \cdot \mathcal{L}(i^*) \\
& + K(f_t^g) \cdot \Big[1\{\beta < K(f_t^g) < 1\} \cdot \mathcal{L}(s(f_t^g) - 1) + 1\{0 < K(f_t^g) \leq \beta\} \cdot \mathcal{L}(\frac{i^* + \bar{\pi}}{1 - \bar{\pi}})\Big] \\
& + (1 - K(f_t^g)) \cdot \Big[1\{\beta < K(f_t^g) < 1\} \cdot \mathcal{L}(\frac{(1 + i^*)(s(f_t^g) - 1)\bar{\pi}}{s(f_t^g) - (1 + i^*)}) \\
& + 1\{0 < K(f_t^g) \leq \beta\} \cdot \mathcal{L}(i^* + \bar{\pi})\Big] . \qquad (2.42)
\end{aligned}
$$

For any given value of $f_t^g \in (\theta - \delta, \theta + \delta)$, opening the market for private loans lowers the aggregate demand for money. The possibility to engage in loans pushes up the opportunity cost of holding cash: domestic residents all find some lucrative businesses yielding higher returns than the foreign bond. For those who expect the peg $\bar{S}$ to remain, by lending to the other crowd, they enjoy an interest rate higher than $(1 + i^*)$; for those who expect the peg $\bar{S}$ to collapse, by borrowing under a high nominal interest rate but accordingly a low real rate, they enjoy the arbitrage from selling pesos short. Therefore, private loan transactions lower the demand for money in the aggregate for any given value of $f_t^g \in (\theta - \delta, \theta + \delta)$.

Further, counter-intuitively the aggregate demand for domestic money can be lower when only a fraction of domestic residents who expect the peg to collapse than when all domestic residents expect so. Precisely, $m_t(f_t^g; \theta, \delta) = \mathcal{L}(i^* + \bar{\pi})$ provided $f_t^g \leq \theta - \delta$; $m_t(f_t^g; \theta, \delta) = \beta \cdot \mathcal{L}(\frac{i^* + \bar{\pi}}{1 - \bar{\pi}}) + (1 - \beta) \cdot \mathcal{L}(i^* + \bar{\pi})$, when $K(f_t^g) = \beta$. Interestingly, the nominal interest rate prevailing in the market is $\frac{i^* + \bar{\pi}}{1 - \bar{\pi}}$ both when $f_t^g \leq \theta - \delta$ and when $K(f_t^g) = \beta$. What makes the difference in the aggregate demand for money is that in the latter case a fraction of domestic residents expect the peg to remain while they expect it to collapse in the former case. Given the same nominal interest rate, a resident perceives a higher opportunity cost of holding domestic money when a devaluation is expected than when it is not.

Due to the observations above, intuitively loan transactions within the private sector arising from heterogeneous beliefs might make a pegged exchange rate vulnerable to speculative attacks. The following two results formalize this idea. I first show that given a distribution of beliefs on $\bar{f}^g$, there exists a situation in which the peg $\bar{S}$ can remain viable permanently provided private loans are not allowed but it can be brought down by speculative attacks at some arbitrary date when private loans are allowed.

Theorem 3. *Domestic residents' beliefs about $\bar{f}^g$ are uniformly distributed over $[\theta - \delta, \theta + \delta] \subset [0, 1]$. $\exists$ a value for $\theta < \mathcal{L}(i^* + \bar{\pi}) + \delta$, such that when collateralized private loans are allowed to trade freely among domestic residents, there exist at least two types of equilibria.*

1. *At all dates all domestic residents expect no devaluation and hold domestic money $\mathcal{L}(i^*)$, and the domestic central bank pegs the exchange rate at $\bar{S}$ at all*

dates.

2. *At at some arbitrary date t, $f_t^g \in (\theta - \delta, \theta)$, and the central bank abandons the peg $\bar{S}$ at date $t+1$.*

Proof. Since $m_t(\theta-\delta; \theta, \bar{\pi}) = \mathcal{L}(i^* + \bar{\pi}) > \theta - \delta$, it is always possible to find a value for θ such that $\exists 0 < \epsilon < \beta$, $\epsilon \cdot \mathcal{L}(\frac{i^*+\bar{\pi}}{1-\bar{\pi}}) + (1-\epsilon) \cdot \mathcal{L}(i^* + \bar{\pi}) < f_t^g$ and $f_t^g = \theta - (1-2\epsilon)\delta < \theta$. □

Second, I show that when $\theta = \mathcal{L}(i^* + \bar{\pi})$ the peg $\bar{S}$ is perpetually viable if domestic residents have the homogeneous belief about $\bar{f}^g$ which coincides with the true value, while the peg $\bar{S}$ can be brought down by speculative attacks as a small enough perturbation of beliefs is introduced, i.e., as δ is small enough.

Theorem 4. *When $\theta = \mathcal{L}(i^* + \bar{\pi})$, $\exists$ a value for δ, such that if domestic residents' beliefs about $\bar{f}^g$ are uniformly distributed over $[\theta - \delta, \theta + \delta] \subset [0, 1]$ and if collateralized private loans are allowed to trade freely among domestic residents, there exist at least two types of equilibria.*

1. *At all dates all domestic residents expect no devaluation and hold domestic money $\mathcal{L}(i^*)$, and the domestic central bank pegs the exchange rate at $\bar{S}$ at all dates.*
2. *At at some arbitrary date t, $f_t^g \in (\theta - \delta, \theta)$, and the central bank abandons the peg $\bar{S}$ at date $t+1$.*

Proof. Since $m_t(\theta - \delta; \theta, \bar{\pi}) = \mathcal{L}(i^* + \bar{\pi}) = \theta$, it is always possible to find a value for δ such that $\exists 0 < \epsilon < \beta$, $\epsilon \cdot \mathcal{L}(\frac{i^*+\bar{\pi}}{1-\bar{\pi}}) + (1-\epsilon) \cdot \mathcal{L}(i^* + \bar{\pi}) < f_t^g$ and $f_t^g = \theta - (1-2\epsilon)\delta < \theta$. □

CHAPTER III

CREDIT DEFAULT SWAPS

A. The model

1. Setup

Consider an economy of two periods, which is inhabited by a continuum of individuals of mass 1. There are one consumption good, and two perfectly divisible assets both of which yield nothing in the 1st period but some exogenous flows of the consumption good in the 2nd period. The difference is: one pays a non-random return rate and the other random. I call the former the risk-free asset denoted by a, (cash and unit is dollar) while the latter the risky asset denoted by k, (house).

I normalize the return rate for the risk-free asset to 1, i.e., holding 1 unit of the risk-free asset in the 1st period gives 1 unit of the consumption good in the 2nd period. The return rate of the risky asset is denoted by a random variable X, and x denotes the realization of X. The aggregate supplies of both assets in the economy are fixed. All individuals are initially endowed with a_0 amount of the risk-free asset and k_0 amount of the risky asset, but there is no endowment of the consumption good in the 1st period.

In addition to the two physical assets mentioned above, I consider loans among individuals collateralized by the risky asset (mortgages). But I only consider loans in which the repayments are expressed in terms of the risk-free asset (cash) and hence I exclude short sales against the risky asset (house). Further, I introduce credit default swaps which reference the collateralized loans. Detailed discussions of these two financial assets are in the succeeding subsections.

Individuals are identical in all aspects expect for the beliefs about X. Heterogeneous beliefs about the return rate of the risky asset give rise to transactions of loans and credit default swaps among individuals. The problem facing a typical individual is to select a portfolio in the 1st period, given its belief about X. Individuals do not consume in the 1st period and the menu for choosing a portfolio includes: the risk-free asset, the risky asset, loans collateralized by the risky asset, and credit default swaps on collateralized loans. In the 2nd period, after the true value of X is revealed, individuals collect proceeds, make payments, and consume. The utility function for a typical individual, denoted by $u(\cdot)$, depends on its consumption in the 2nd period only. I make the standard assumptions: $u'(\cdot) > 0$ and $u''(\cdot) \leq 0$.

2. Collateralized loans

Loans are all initiated in the 1st period and mature in the 2nd period. If an individual lends, he pays certain amount of the risk-free asset in the 1st period and is supposed to receive a promised amount of the risk-free asset as the repayment in the 2nd period; if the individual borrows, he receives certain amount of the risk-free asset instead and meanwhile is required to hold certain amount of the risky asset as the collateral in the 1st period, and in the 2nd period he is supposed to repay the promised amount of the risk-free asset. When the loan is not repaid, the collateral is seized to pay off the loan.

A collateralized loan is defined as a pair, (R, c), which states the promised repayment rate and the collateral rate. Table II gives the definition for each contract terms.

For example, consider loan $(1.2, 1.1)$ and suppose the notional value is 100. $R = 1.2$ means that 120 units of the risk-free asset needs to repaid in the 2nd period. $c_k = 1.1$ means the risky asset pledged as the collateral is worth 110 units of the

Table II. Two contract terms in a loan

Variable		
R	the Gross Interest Rate	
	Formula:	promised amount of the risk-free asset/ notional value
c	the Collateral Rate	
	Formula:	value of the risky asset posted as the collateral/ notional value

risk-free asset in the 1st period.

Individuals borrow and lend not in a bilateral way but in Walrasian markets. For each loan, given its associated price, individuals submit their orders of borrowing or lending to a Walrasian auctioneer. For example, if individuals want to borrow or to lend under loan $(1.2, 1.1)$, they need to tell the auctioneer how much they want to borrow or to lend. The notional value is the number that individuals use to communicate with the auctioneer about the quantity to borrow or to lend. If a borrowing order is placed under loan $(1.2, 1.1)$ of a notional value 100, the auctioneer writes -100 in its calculation of market clearing. If a lending order is placed of a notional value 150, the auctioneer adds $+150$. The market for loan $(1.2, 1.1)$ is cleared, when after summing up the numbers from all orders the auctioneer gets zero.

The price of a loan is the amount of the risk-free asset per notional value to pay for those who want to lend (or to receive for those who want to borrow) in the 1st period. For example, if the price of loan $(1.2, 1.1)$ is 0.5 units of the risk-free asset per notional value, and if an individual wants to lend under loan $(1.2, 1.1)$ of a notional value 100, he needs to pay 50 units of the risk-free asset. Formally, there is a mapping which associates a loan with a price,

$$p(R,c) : \Re_+^2 \rightarrow \Re_+ \ , \tag{3.1}$$

$p(R,c)$ is the price for loan (R,c). The auctioneer announces the price of a loan and adjusts it until the market for the loan is cleared. After a market clearing price is reached, the auctioneer collects payments from those who place a lending order and distribute them to those who place a borrowing order in the 1st period and collects repayments from borrowers and distribute them to lenders in the 2nd period. The promised repayment of a loan does not necessarily coincide with the actual repayment. Under the collateral mechanism, borrowers will repay the loan only when the collateral is worth more than the amount due, i.e., whenever

$$R \leq \frac{X}{q} \cdot c \ , \tag{3.2}$$

the loan will be paid off, otherwise borrowers default and the collateral is seized. Note whether or not the loan is paid off is contingent on the value of X. $r_l(X,q;R,c)$ denotes the actual return rate for loan (R,c), and

$$r_l(X,q;R,c) = \min\{R, (\frac{X}{q} \cdot c)\} \ . \tag{3.3}$$

Note $r_l(X,q;R,c)$ is continuous and homogeneous of degree one with respect to (R,c). So is $p(R,c)$, if certain types of arbitrage activities are allowed. Without loss of generality, I assume individuals restrict their choices to the set of loans the price of which is 1. Ω denotes this set,

$$\Omega \equiv \{(R,c) \in \Re_+^2 \parallel p(R,c) = 1\} \ . \tag{3.4}$$

Ω represents all loans available in the market.

3. Credit default swaps

For each loan, there is a corresponding credit default swap (CDS). In the 1st period, if an individual buys a CDS on a particular loan (R, c) of a notional value, he pays certain amount of the risk-free asset as the premium in the 1st period; he receives the premium if the individual sells the CDS. In the 2nd period only if loan (R, c) is not paid off, i.e., when $R > \frac{X}{q} \cdot c$, the individual pays as a CDS seller (or receives as a CDS buyer) the difference between the promised loan repayment and the value of the collateral, i.e., $R - (\frac{X}{q} \cdot c)$ units of the risk-free asset per notional value. The spread of a CDS is defined as the amount of the risk-free asset per notional value to pay for those who buy the CDS (or to receive for those who sell the CDS) in the 1st period. Formally,

$$\pi(R, c) : \Re^2_+ \to \Re_+ \ , \tag{3.5}$$

$\pi(R, c)$ denotes the spread of the CDS on loan (R, c). Take the example of loan $(1.2, 1.1)$, and suppose $q = 1$, and $\pi(1.2, 1.1) = 0.2$. If an individual buys the CDS on loan $(1.2, 1.1)$ of a notional value 100, the individual needs to pay 20 units of the risk-free asset in the 1st period as the premium. If $x = 2$, loan $(1.2, 1.1)$ is paid off and the individual gets nothing in this contingency. If $x = 1$, loan $(1.2, 1.1)$ is not paid off and the individual receives 10 $(= 120 - 110)$ units of the risk-free asset in the 2nd period. $r_s(X, q; R, c)$ denotes the actual return rate for the CDS on loan (R, c), and

$$r_s(X, q; R, c) = \max\{R - (\frac{X}{q} \cdot c), 0\} \ . \tag{3.6}$$

I make two assumptions on CDS trading, which have important consequences throughout the paper. The first assumption makes sure that all credit default swaps are backed by a sufficient amount of collateral so that in any contingency CDS sellers are able to deliver promised payoffs.

Assumption 1. *Sellers of credit default swaps need to post a sufficient amount of collateral so that the promised payments made on a credit default swap will be made in every contingency, i.e., for a credit default swap on loan (R, c), a seller needs to hold R units of the risk-free asset per notional value in the 1st period.*

The immediate consequence of this assumption is that lending under any loan and meanwhile buying the corresponding CDS of the same notional value give a risk-free return rate. Formally, $\forall x, q$, and (R, c),

$$r_l(x, q; R, c) + r_s(x, q; R, c) = R\ . \tag{3.7}$$

Since Assumption 1 requires CDS sellers to post a sufficient amount of collateral which might be unnecessary for certain contingencies. The second assumption allows CDS sellers to economize on collateral: the abundant part of the collateral used for credit default swaps can be used for corresponding loans. The rationale for the second assumption comes from the fact that the collateral posted for the credit default swap on loan (R, c) of a notional value 1 minus the promised payment is $R - r_s(X, q; R, c) = r_l(X, q; R, c)$, which can be used as an equivalent substitute for the risky asset as collateral for loan (R, c).

Assumption 2. *The remnant of the collateral used to back up a credit default swap on a loan can be used as collateral for the same loan and it is viewed as an equivalent substitute for the risky asset.*

Under two assumptions above and if certain types of arbitrage activities are allowed, the sum of the spread of a CDS and the price of the corresponding loan should be equal to the promised repayment rate on the loan, i.e., $\forall(R,c) \in \Re_+^2$,

$$\pi(R,c) = R - p(R,c) \ . \tag{3.8}$$

The implications of the equality (3.8), Assumption 1, and Assumption 2 are profound. Several comments are in order. First, if an individual wants to lend under a loan and buy the credit default swap on the same loan, the individual can in fact obtain the same payoffs in all contingencies by lending less and holding some additional amount of the risk-free asset. For example, consider lending under loan $(1.05, 1.1)$ of a notional value 10000 and buying the credit default swap on loan $(1.05, 1.1)$ of a notional value 1000. Regardless of the value of x, this operation yields the same payoffs as lending under loan $(1.05, 1.1)$ of a notional value 9000 and meanwhile holding 1050 units of the risk-free asset.

Second, if an individual wants to sell a CDS on a loan, the individual can obtain an equivalent payoffs schedule by lending under the corresponding loan. This is due to equality (3.8) and (3.7).

Third, as I examine the role that credit default swaps play as insurance only, i.e., to hedge default risks, the first fact that buying credit default swaps to alleviate potential losses from loan defaults is equivalent with lending less and holding additional amounts of the risk-free asset and the second fact that selling credit default swaps is equivalent with lending under the corresponding loans, constitute the core reasons why the introduction of credit default swaps only as insurance has neither effect on the price of the risky asset nor the market for collateralized loans. Note these two facts are independent of assumptions on individuals' beliefs on X.

4. Individual problem

Since individuals are different only by their beliefs about X, I analyze individuals' choices in the 1st period in a generic way. I assume that a typical individual has a subjective probability distribution regarding X and the cumulative distribution function is denoted by $F(\cdot)$. The objective for a typical individual is to maximize

$$\int u(z)dF(x) \ , \tag{3.9}$$

where z denotes the consumption in the 2nd period,

$$z \ = x \cdot k + a + \sum_{\Omega} \left[r_l(x, q; R, c) \cdot L(R, c) + r_s(x, q; R, c) \cdot S(R, c) \right] \ , \tag{3.10}$$

where $L(R, c)$ denotes the notional value of loan (R, c) to borrow or to lend (If $L(R, c) < 0$ it denotes borrowing); $S(R, c)$ denotes the notional value of CDS on loan (R, c) to buy or to sell (If $S(R, c) < 0$ it denotes selling), subject to

$$q \cdot k + a + \sum_{\Omega} \left[1 \cdot L(R, c) + \pi(R, c) \cdot S(R, c) \right] = q \cdot k_0 + a_0 \ , \tag{3.11}$$

and two collateral constraints,

$$-\sum_{\Omega} R \cdot \min\{S(R, c)\} \leq a \ , \tag{3.12}$$

$$-\sum_{\Omega} \left[c \cdot \min\big\{ (\min\{L(R, c), 0\} - \min\{S(R, c), 0\}), 0 \big\} \right] \leq q \cdot k \ . \tag{3.13}$$

Note (3.12) is due to Assumption 1, and (3.13) considers $S(\cdot)$ is due to Assumption 2. To write out the Lagrange, where λ is the lagrange multiplier for (3.11), ν for (3.12), and μ for (3.13).

Lagrange:

$$
\begin{aligned}
& u(z) \\
& + \lambda \cdot \Big(qk_0 + a_0 - q \cdot k - a - \sum_{\Omega} \big[L(R,c) + \pi(R,c)S(R,c)\big]\Big) \\
& + \nu \cdot \Big(a + \sum_{\Omega} R \cdot \min\{S(R,c), 0\}\Big) \\
& + \mu \cdot \Big(q \cdot k + \sum_{\Omega} \big[c \cdot \min\big\{(\min\{L(R,c), 0\} - \min\{S(R,c), 0\}), 0\big\}\big]\Big) \,. \qquad (3.14)
\end{aligned}
$$

As I have illustrated intuitively, under Assumption 1, Assumption 2, and equality 3.8, lending under a loan and meanwhile buying a CDS on the same loan of the same notional value should be equivalent with holding the risk-free asset, and selling a CDS on a loan is equivalent with lending under the same loan. The following lemma formalizes these results. And note this lemma holds true regardless of the assumptions on beliefs about X.

Lemma 6. *Assumption 1-2 and equality 3.8 hold. Suppose $\{k, a, L(\cdot), S(\cdot)\}$ maximizes (3.9) subject to (3.11)-(3.13). If a particular loan $(R_0, c_0) \in \Omega$ such that $L(R_0, c_0) \neq 0$ and $S(R_0, c_0) \neq 0$, and then $\exists\{\tilde{k}, \tilde{a}, \tilde{L}(\cdot), \tilde{S}(\cdot)\}$, which is a maximizer too but $\tilde{L}(R_0, c_0) \cdot \tilde{S}(R_0, c_0) = 0$.*

By the lemma above, I can concentrate on choices by a typical individual in which $\forall(R, c) \in \Re^2_+$, $L(R, c) \cdot S(R, c) = 0$. In below, I state the necessary conditions for the degenerate case of $F(\cdot)$, i.e., the individual has a point estimate of X. Therefore, the objective to maximize (3.9) is equivalent to maximize (3.10).

Lemma 7. *Suppose $\{k, a, L(\cdot), S(\cdot)\}$ maximizes (3.10) subject to (3.11)-(3.13), and*

$\forall (R,c) \in \Re^2_+$, $L(R,c) \cdot S(R,c) = 0$. The conditions in below must be satisfied.

$$x - \lambda \cdot q + \mu \cdot q \leq 0 \ , \tag{3.15}$$

where the equality holds if $k > 0$;

$$1 - \lambda + \nu \leq 0 \ , \tag{3.16}$$

where the equality holds if $a > 0$; $\forall (R,c) \in \Omega$, if $L(R,c) \neq 0$, $S(R,c) = 0$,

$$r_l(x,q;R,c) - \lambda + \mu \cdot c \cdot 1\{L(R,c) < 0\} = 0 \ , \tag{3.17}$$

$$r_s(x,q;R,c) - \pi \cdot \lambda \leq 0 \ , \tag{3.18}$$

$$r_s(x,q;R,c) - \pi \cdot \lambda + \nu \cdot R - \mu \cdot c \cdot 1\{L(R,c) < 0\} \geq 0 \ , \tag{3.19}$$

if $L(R,c) = 0$, $S(R,c) \neq 0$,

$$r_l(x,q;R,c) - \lambda \leq 0 \ , \tag{3.20}$$

$$r_l(x,q;R,c) - \lambda + \mu \cdot c \cdot 1\{S(R,c) > 0\} \geq 0 \ , \tag{3.21}$$

$$r_s(x,q;R,c) - \pi \cdot \lambda + \nu \cdot R \cdot 1\{S(R,c) < 0\} = 0 \ , \tag{3.22}$$

if $L(R,c) = 0$, $S(R,c) = 0$,

$$r_l(x,q;R,c) - \lambda \leq 0 \ , \tag{3.23}$$

$$r_l(x,q;R,c) - \lambda + \mu \cdot c \geq 0 \ , \tag{3.24}$$

$$r_s(x,q;R,c) - \pi \cdot \lambda \leq 0 \ , \tag{3.25}$$

$$r_s(x,q;R,c) - \pi \cdot \lambda + \nu \cdot R \geq 0 \ . \tag{3.26}$$

B. Equilibrium

An equilibrium is defined as a state in which given $p(\cdot)$ and q all markets are cleared in the 1st period. In below, I first examine the effect of introducing credit default swaps only as insurance. The following result does not depend on how individuals' beliefs about X are specified. To restrict the uses of credit default swaps to insurance only, the possibility of buying credit default swaps with no investment in the corresponding loans must be excluded. The following assumption states this restriction formally.

Assumption 3. *It is not allowed to purchase credit default swaps on any loan of a notional value exceeding the actual holding of this loan. Precisely,* $\forall (R,c) \in \Re^2_+$, $L(R,c) \geq S(R,c)$ *provided* $S(R,c) > 0$.

I show that the introduction of credit default swaps only as insurance has no effect on the price of the risky asset and the market for collateralized loans in equilibrium. As I have mentioned earlier, the core reasons for the zero-effect result stem from two facts: under Assumption 1-2 and equality 3.8, first buying credit default swaps to alleviate potential losses from loan defaults is equivalent with lending less and holding additional amounts of the risk-free asset, and second selling credit default swaps is equivalent with lending under the corresponding loans. The following theorem states the result formally.

Theorem 5. *Under Assumption 1-2 and equality 3.8, if* $\{q^*, p^*(\cdot)\}$ *is an equilibrium when credit default swaps are not allowed, it must be an equilibrium when credit default swaps are allowed but the uses of them are restricted under Assumption 3, vise versa.*

The theorem above deals with the effect of introducing credit default swaps only as insurance. In below, I assume that credit default swaps not only can be used to hedge default risks but also to bet against loans collateralized by the risky asset.

Moreover, I make a concrete specification of individuals' beliefs about X. Precisely, I assume that the economy is divided by two groups: "optimists" and "pessimists", both of which have a point estimate of X, as Table III illustrates. I use the notations in Table IV to differentiate variables associated with two separate groups.

Table III. Specification of beliefs

groups	point estimate of X	population
optimists	x_h	$1-\theta$
pessimists	x_l	θ

Table IV. Notations

variable	optimists	pessimists
a:	a^o	a^p
k:	k^o	k^p
$L(\cdot)$:	$L^o(\cdot)$	$L^p(\cdot)$
$S(\cdot)$:	$S^o(\cdot)$	$S^p(\cdot)$
λ:	λ^o	λ^p
μ:	μ^o	μ^p
ν:	ν^o	ν^p

When both collateralized loans and credit default swaps are not allowed, in the 1st period there are only two assets, the risky and risk-free asset. It is easy to derive the equilibrium value for q in this case.

$$\begin{cases} q = \frac{1-\theta}{\theta} \cdot \frac{a_0}{k_0} & \text{, if } \frac{\frac{a_0}{k_0}}{x_h + \frac{a_0}{k_0}} < \theta < \frac{\frac{a_0}{k_0}}{x_l + \frac{a_0}{k_0}}; \\ q = x_l & \text{, if } \theta \geq \frac{\frac{a_0}{k_0}}{x_l + \frac{a_0}{k_0}}; \\ q = x_h & \text{, if } \theta \leq \frac{\frac{a_0}{k_0}}{x_h + \frac{a_0}{k_0}}. \end{cases} \tag{3.27}$$

From this point on, I assume that loans collateralized by the risky asset are always allowed, and I solve for $\{q, p(\cdot)\}$ in equilibrium analytically when credit default swaps are banned and are allowed respectively, and then compare the differences. Before heading forward, I provide a result, useful later and independent of whether or not credit default swaps are allowed. The following lemma states that the price of the risky asset should lie in between two polars of opinion among individuals.

Lemma 8. *In equilibrium, when collateralized loans are allowed, regardless of whether or not trading credit default swaps is allowed, the price of the risky asset,* $q \in [x_l, x_h]$.

Proof. Step1: suppose $q > x_h$. $\exists$ an individual in the economy who holds the risky asset. $q > x_h$ implies that he must borrow under some loan $(R, c) \in \Omega$. To make the loan (R, c) attractive, it must be $c > 1$, which in turn implies no one in the economy wants to borrow under the loan (R, c). Step2: suppose $q < x_l$. $\exists$ an individual in the economy who holds the risk-free asset. $q < x_l$ implies that he must sell a CDS on a loan $(R, c) \in \Omega$ and thus $\min\{R, \frac{x_l}{q}c\} > 1$, which implies no one in the economy wants to buy CDS on the loan (R, c). $\square$

1. Without credit default swaps

To solve for $\{q, p(\cdot)\}$, first I derive the equilibrium conditions on q and Ω, given how individuals transact collateralized loans. There are only three cases: no loan transaction at all, pessimists borrowing from optimists, or optimists borrowing from pessimists.

Lemma 9. *When trading credit default swaps is not allowed, if in equilibrium there is no loan transaction among individuals, it must be that*

$$\Omega = \{(R, c) \| r_l(x_h, q; R, c) = 1\} , \tag{3.28}$$

and $q = x_h \leq \frac{1-\theta}{\theta} \frac{a_0}{k_0}$.

Proof. Optimists must hold non-zero risky asset. Since there is no borrowing and lending, $\mu^o = 0$. Hence $\Omega = \{(R, c) \| r_l(x_h, q; R, c) = \frac{x_h}{q}\}$. Since $r_l(x_l, q; \frac{x_h}{q}, \frac{x_h}{x_l}) = \frac{x_h}{q}$, it must be that $\frac{x_h}{q} = 1$, otherwise no one holds the risk-free asset. When $q = x_h$, pessimists must hold zero risky asset. Hence the market clearing condition requires that $qk_0 + a_0 \geq q\frac{k_0}{1-\theta}$, which implies $x_h \leq \frac{1-\theta}{\theta} \frac{a_0}{k_0}$. □

Lemma 10. *When trading credit default swaps is not allowed, if in equilibrium pessimists borrow from optimists under some loan* $(R, c) \in \Omega$*, it must be that* $c = 1$*,* $R \geq 1$*,* $q = x_h$*, and* Ω *satisfies (3.28).*

Proof. Suppose optimists lend to pessimists under loan $(R, c) \in \Omega$. It must be that, $r_l(x_h, q; R, c) \geq \max\{1, \frac{x_h}{q}\}$. Step1: $R < \frac{x_l}{q}c$. It must be that $R = 1$ and $q = x_l$. But meanwhile, $r_l(x_h, q; R, c) = R = 1 \geq \frac{x_h}{q}$, and thus $x_l \geq x_h$. Contradiction. Step2: $R \geq \frac{x_l}{q}c$. It must be that $c = 1$. Hence $r_l(x_h, q; R, c) = \lambda^o = \frac{x_h}{q}$, which implies $\mu^o = 0$ and $R \geq \frac{x_h}{q}$. Hence $\Omega = \{(R, c) \| r_l(x_h, q; R, c) = \frac{x_h}{q}\}$. Since $r_l(x_l, q; \frac{x_h}{q}, \frac{x_h}{x_l}) = \frac{x_h}{q}$, it must be that $\frac{x_h}{q} = 1$, otherwise no one holds the risk-free asset. □

Lemma 11. *When credit default swaps are not allowed, if in equilibrium optimists borrow from pessimists under loan* $(R, c) \in \Omega$*, it must be that* $r_l(x_l, q; R, c) = 1$ *and* $1 = R \leq \frac{x_l}{q}c$*. Either* $\mu^o = 0$*,* $q = x_h$*, and* Ω *satisfies (3.28) or* $\mu^o > 0$*,* $q < x_h$*,* $R = \frac{x_l}{q}c$*, and* Ω *is not unique and a possible solution is:*

$$\Omega = \{(R,c)\|R > \frac{x_h}{q}, c = 1\} \cup \{(R,c)\|R = 1, c > \frac{q}{x_l}\} \\ \cup \{(R,c)\|(R,c) = \beta \cdot (\frac{x_h}{q}, 1) + (1-\beta)\cdot(1, \frac{q}{x_l}), \beta \in [0,1]\} \ . \tag{3.29}$$

Proof. Suppose in the equilibrium that optimists borrow from pessimists under loan $(R, c) \in \Omega$ where $R > \frac{x_l}{q}c = 1$. If $R \geq \frac{x_h}{q}c$, it must be that $c = 1$, which implies $q = x_l$ and thus for pessimists $\mu^p = 0$. Hence $\Omega = \{(R,c)\|r_l(x_l, q; R, c) = 1\}$. And thus loan $(1, 1) \in \Omega$. Optimists want to borrow to infinity under loan$(1, 1)$. Contradiction. If $\frac{x_l}{q}c < R < \frac{x_h}{q}c$, $\exists \epsilon > 0$ such that $R - \epsilon > \frac{x_l}{q}c$, to make pessimists indifferent between loan (R, c) and $(R - \epsilon, c)$, $p(R - \epsilon, c) = 1$, which implies that optimists prefer loan $(R - \epsilon, c)$ over (R, c). Contradiction. □

The following theorem presents how the equilibrium changes as θ increases. The equilibrium price of the risky asset in general falls as θ increases, which is illustrated in Fig. 6. When

$$\theta \leq \frac{x_l + \frac{a_0}{k_0}}{x_h + \frac{a_0}{k_0}} \ , \tag{3.30}$$

$q = x_h$, the price of the risky asset reaches a level so that optimists are indifferent between the risky and risk-free asset, indifferent between borrowing and lending for all loans in Ω. Fig. 7 depicts Ω in this case. Since in this case the price of the risky asset in the 1st period is so high that pessimists would not hold the risky asset. Hence optimists as the counterpart need to absorb the entire supply of the risky asset in the economy.

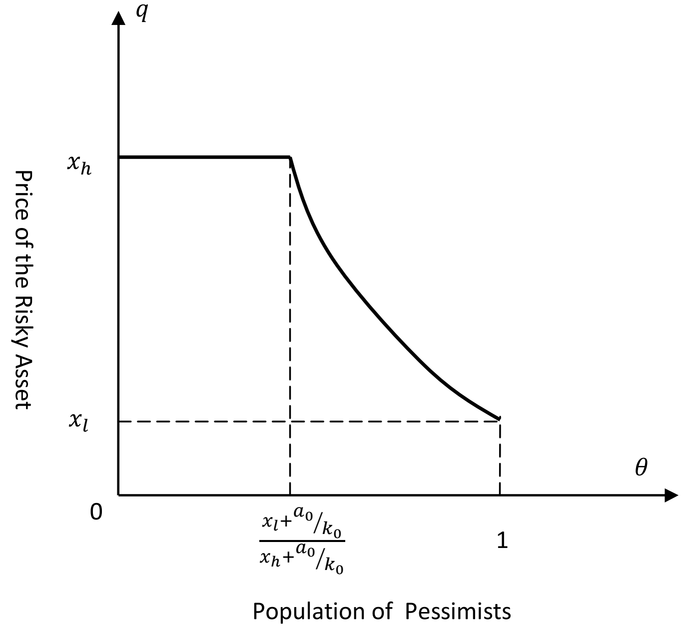


Fig. 6. q when credit default swaps are prohibited.

When

$$\theta \leq \frac{\frac{a_0}{k_0}}{x_h + \frac{a_0}{k_0}} \ , \tag{3.31}$$

optimists can absorb the entire supply of the risky asset without borrowing. But when

$$\frac{\frac{a_0}{k_0}}{x_h + \frac{a_0}{k_0}} < \theta \leq \frac{x_l + \frac{a_0}{k_0}}{x_h + \frac{a_0}{k_0}} \ , \tag{3.32}$$

optimists finance the purchase of the entire supply of the risky asset through the collateralized loans. Moreover, in this case, optimists can still provide plenty of collateral.

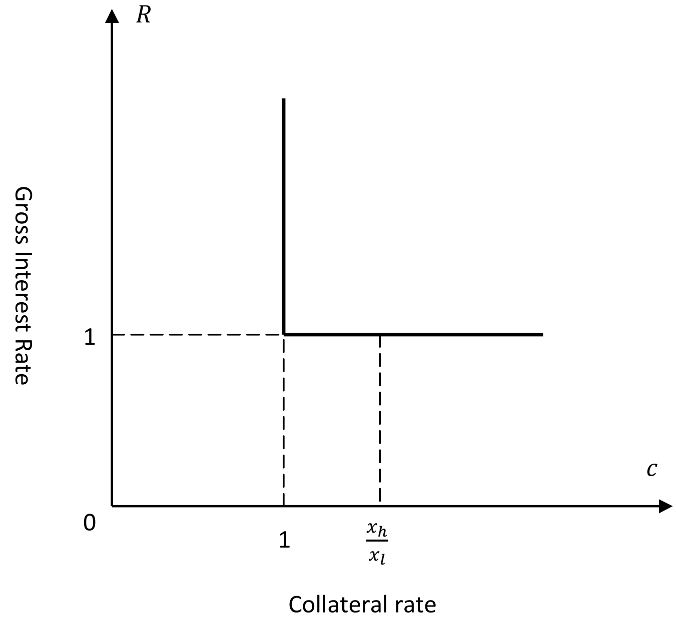


Fig. 7. Ω when credit default swaps are prohibited and $\theta \leq \frac{x_l + \frac{a_0}{k_0}}{x_h + \frac{a_0}{k_0}}$.

As the population of pessimists rises further, i.e., as

$$\theta > \frac{x_l + \frac{a_0}{k_0}}{x_h + \frac{a_0}{k_0}} \ , \tag{3.33}$$

a single individual in the optimistic crowd needs to borrow more while borrowing is restricted by the amount of collateral, and hence the price of the risky asset and the collateral rate have to fall. Fig. 8 depicts Ω in this case.

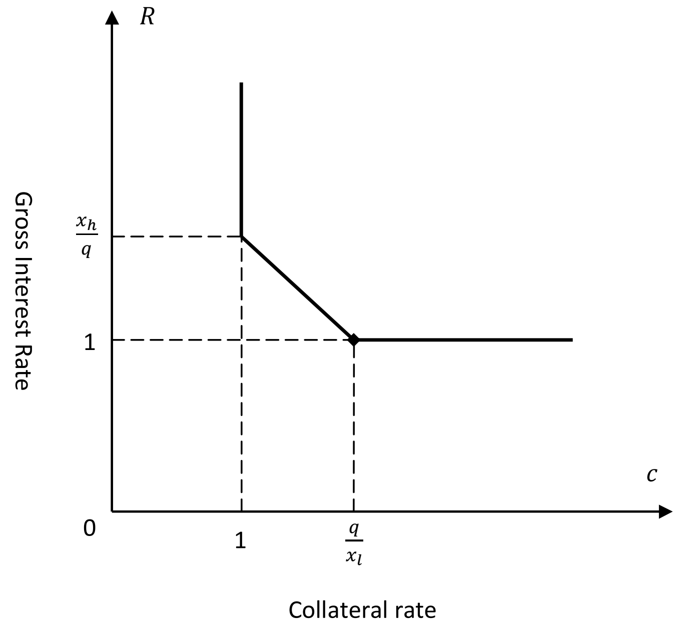


Fig. 8. Ω when credit default swaps are prohibited and $\theta > \frac{x_l + \frac{a_0}{k_0}}{x_h + \frac{a_0}{k_0}}$.

Theorem 6. *When the market for credit default swaps is not open, the equilibrium changes as follows.*

- *If $x_h - \frac{x_l}{\theta} \leq \frac{1-\theta}{\theta}\frac{a_0}{k_0}$, $q = x_h$, Ω satisfies (3.28). Moreover, if $x_h \leq \frac{1-\theta}{\theta}\frac{a_0}{k_0}$, the quantities of loans traded are indeterminant and it is possible there are no loan transactions at all. If there are loan transactions, possibilities are: optimists borrow from pessimists under loans $(1, c)$ with $c \geq \frac{x_h}{x_l}$; pessimists borrow from optimists under loans $(R, 1)$ with $R \geq 1$. When $x_h - \frac{x_l}{\theta} \leq \frac{1-\theta}{\theta}\frac{a_0}{k_0} < x_h$, optimists must borrow from pessimists under loans $(1, c)$ with $c \geq \frac{x_h}{x_l}$.*
- *If $\frac{1-\theta}{\theta}\frac{a_0}{k_0} < x_h - \frac{x_l}{\theta}$, $q = \frac{x_l}{\theta} + \frac{1-\theta}{\theta}\frac{a_0}{k_0}$, the only loan transaction is that optimists borrow from pessimists under loan $(1, \frac{q}{x_l})$, and Ω satisfies (3.29). Moreover, if*

$\frac{x_l}{\theta} + \frac{1-\theta}{\theta}\frac{a_0}{k_0} < x_l(2 - \frac{x_l}{x_h})$, *another possible solution for* Ω *is:*

$$\Omega = \{(R, c) \| r_l(x_l, q; R, c) = 1\} . \tag{3.34}$$

Proof. Step1: suppose $x_h - \frac{x_l}{\theta} \leq \frac{1-\theta}{\theta}\frac{a_0}{k_0}$. It must be that $\mu^o = 0$. $\mu^o > 0$ implies that optimists borrow from pessimists under loan $(1, \frac{q}{x_l})$. Hence $x_h > q > x_l$. To clear the market for the risky asset, a typical optimist should hold the risky asset $\frac{k_0}{1-\theta}$ and borrow $a_0 - \frac{\theta}{1-\theta}qk_0$. By $\mu^o > 0$, it must be that $\frac{1-\theta}{\theta}\frac{a_0}{k_0} + \frac{x_l}{\theta} = q$. Hence $q = x_h$. Contradiction. Step2: suppose $x_h - \frac{x_l}{\theta} > \frac{1-\theta}{\theta}\frac{a_0}{k_0}$. It must be that $\mu^o > 0$. $\mu^o = 0$ implies $q = x_h$ and $\frac{1-\theta}{\theta}\frac{a_0}{k_0} + \frac{x_l}{\theta} \leq x_h$. Contradiction. □

One implication of the theorem above is that in general borrowing and lending among individuals push up the price of the risky asset. Intuitively, with the aid of loans collateralized by the risky asset, optimists can purchase the risky asset through borrowing, which as a result lifts up the demand for the risky asset in general. But there is one exception. As the population of pessimists is small enough, precisely, i.e., as $\theta < \frac{a_0/k_0}{x_h + a_0/k_0}$, opening up the market for collateralized loans does not alter the equilibrium price of the risky asset. To see the point, I graph the equilibrium price of the risky asset against the population of pessimists when both collateralized loans are allowed and are prohibited. Fig. 9 illustrates the point for the case where $x_l^2 + 2x_l\frac{a_0}{k_0} < x_h\frac{a_0}{k_0}$.

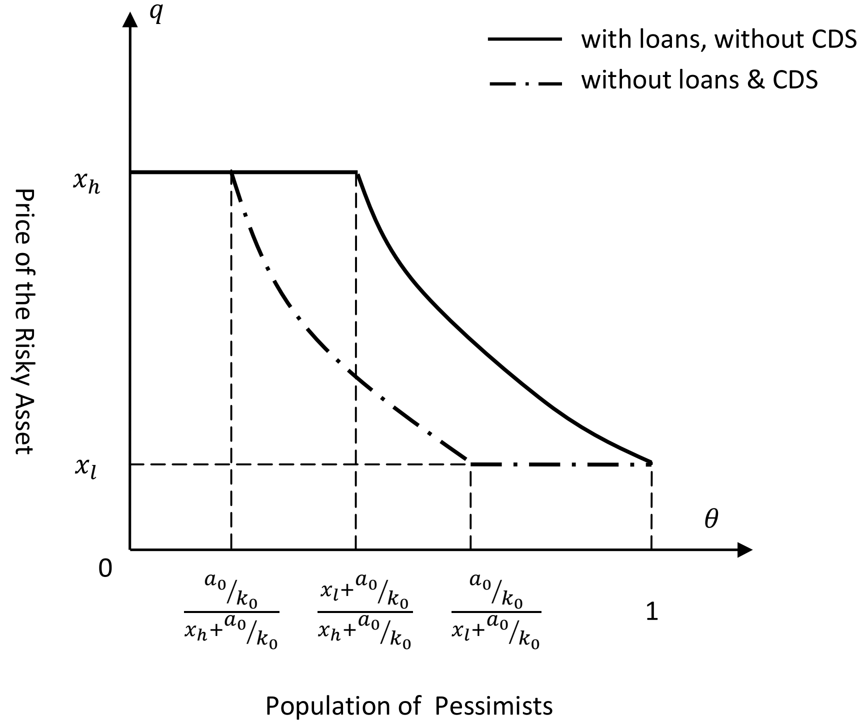


Fig. 9. q in equilibrium when collateralized loans are prohibited and are allowed.

2. With credit default swaps

To solve for $\{q, p(\cdot)\}$, I derive the conditions on q and Ω, and how individuals possibly transact loans and credit default swaps, given the values for λ^o and λ^p. The following lemma guarantees that a restriction on individuals' choices does not alter the equilibrium.

Lemma 12. *Under Assumption 1-2 and equality 3.8, if* $\{q^*, p^*(\cdot)\}$ *is an equilibrium when credit default swaps are allowed, it must be an equilibrium when credit default swaps are allowed but for all individuals* $\forall(R,c) \in \Re^2_+,\ L(R,c) \cdot S(R,c) = 0$*, vise versa.*

The following lemma states that it is impossible that in equilibrium individuals neither transact loans nor credit default swaps.

Lemma 13. *When credit default swaps are allowed, it is impossible in equilibrium that* $\lambda^o = \frac{x_h}{q}$ *and* $\lambda^p = 1$*. As a result, it is impossible that individuals neither transact loans nor credit default swaps in equilibrium.*

Proof. Suppose in an equilibrium, $\lambda^o = \frac{x_h}{q}$ and $\lambda^p = 1$. By Lemma 7 $\mu^o = 0$ and $\nu^p = 0$. And thus $\forall (R,c) \in \Omega$, $r_l(x_h, q; R, c) = \frac{x_h}{q}$ and $r_l(x_l, q; R, c) = 1$. Contradiction. Note $q \in [x_l, x_h]$ in equilibrium by Lemma 8. □

$\lambda^p > 1$ and $\lambda^o > \frac{x_h}{q}$ imply that pessimists and optimists respectively are involved in some businesses which can earn a higher return than holding both the risky and risk-free asset. Note by Lemma 8, $q \in [x_l, x_h]$ in equilibrium.

When $\lambda^o = \frac{x_h}{q}$ and $\lambda^p > 1$, the following lemma states that Ω should be like the one shown in Fig. 10. In this case optimists are indifferent between the loans in Ω and the risky asset while pessimists perceive it lucrative to bet against loan $(\frac{x_h}{q}, 1)$ which they expect to go bad in the 2nd period. It is also possible in this case that optimists borrow from pessimists under loans $(\frac{x_h}{q}, c)$ with $c \geq \frac{x_h}{x_l}$ and that pessimists borrow from optimists under loans $(R, 1)$ with $R \geq \frac{x_h}{q}$.

Lemma 14. *When credit default swaps are allowed, if in equilibrium* $\lambda^o = \frac{x_h}{q}$ *and* $\lambda^p > 1$, $\frac{x_h^2}{2x_h - x_l} \leq q < x_h$,

$$\Omega = \{(R,c) \| r_l(x_h; q, R, c) = \frac{x_h}{q}\} \ , \tag{3.35}$$

and the possible transactions are: pessimists buy from optimists the credit default swap on loan $(\frac{x_h}{q}, 1)$*, optimists borrow from pessimists under loans* $(\frac{x_h}{q}, c)$ *with* $c \geq \frac{x_h}{x_l}$*, and pessimists borrow from optimists under loans* $(R, 1)$ *with* $R \geq \frac{x_h}{q}$*.*

Proof. $\lambda^o = \frac{x_h}{q}$ implies $\mu^o = 0$ and thus Ω satisfies (3.35). $\frac{x_h^2}{2x_h - x_l} > q$ implies that loan $(\frac{x_h}{q}, \frac{x_h}{x_l}) \in \Omega$ dominates the risk-free asset in return and that there are no transactions of CDS. Hence no one holds the risk-free asset. Contradiction. $q = x_h$ implies the CDS on loan $(\frac{x_h}{q}, 1)$ costs nothing to buy while pessimists expect loan $(\frac{x_h}{q}, 1)$ to go bad and hence buy the corresponding CDS to infinity. Contradiction. $x_h^2/(2x_h - x_l) = q$ implies that for pessimists buying CDS on loan $(\frac{x_h}{q}, 1)$ has the same return rate as lending under loans $(\frac{x_h}{q}, c)$ with $c \geq \frac{x_h}{x_l}$. For optimists, loans in Ω are indifferent. Loans $(R, 1)$ with $R \geq \frac{x_h}{q}$ are equivalent with the risky asset for optimists except that the loans can not be used as collateral. Borrowing under loans $(R, 1)$ with $R \geq \frac{x_h}{q}$ is of no real substance for pessimists. $\square$

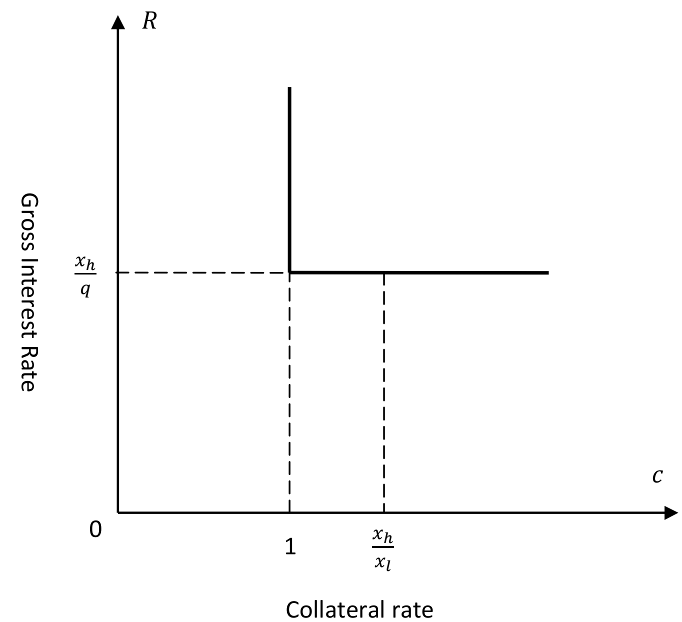


Fig. 10. Ω when credit default swaps are allowed and $\lambda^o = \frac{x_h}{q}$, $\lambda^p > 1$.

When $\lambda^o > \frac{x_h}{q}$ and $\lambda^p = 1$, the following lemma states that Ω should be like the one shown in Fig.11. In this case pessimists are indifferent between the loans in Ω

and the risk-free asset while optimists perceive it lucrative to buy the risky asset on margin under loan $(1, \frac{q}{x_l})$. It is also possible that pessimists buy from optimists the credit default swaps on loans $(R, \frac{q}{x_l})$ with $R \geq \frac{x_h}{x_l}$.

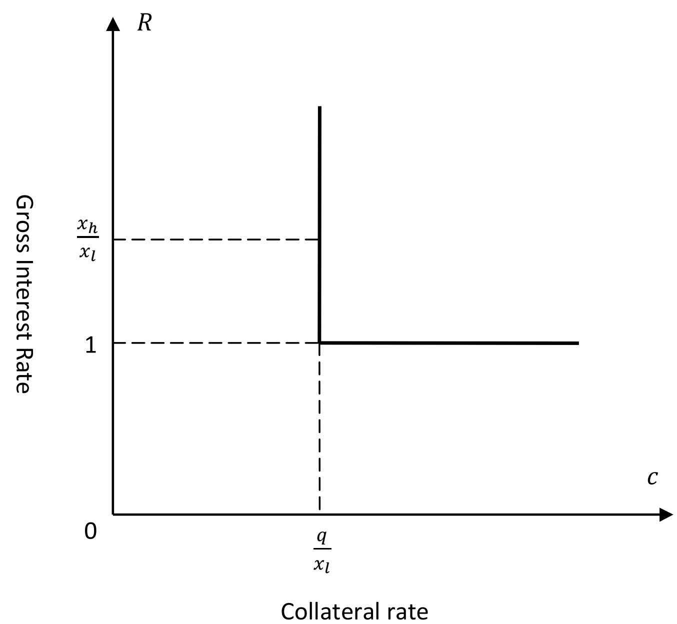


Fig. 11. Ω when credit default swaps are allowed and $\lambda^o > \frac{x_h}{q}$, $\lambda^p = 1$.

Lemma 15. *In equilibrium, if* $\lambda^o > \frac{x_h}{q}$ *and* $\lambda^p = 1$, Ω *satisfies (3.34),* $x_l < q \leq x_l(2 - \frac{x_l}{x_h})$, *and the possible transactions are: optimists borrow from pessimists under the loan* $(1, \frac{q}{x_l})$ *and pessimists buy from optimists the credit default swaps on loans* $(R, \frac{q}{x_l})$ *with* $R \geq \frac{x_h}{x_l}$.

Proof. $\lambda^p = 1$ implies $\nu^p = 0$ and thus Ω satisfies (3.34). $q > x_l(2 - \frac{x_l}{x_h})$ implies that loan $(\frac{x_h}{x_l}, \frac{q}{x_l}) \in \Omega$ dominates the risky asset in return and that there are no loan transactions. Hence no one holds the risky asset. Contradiction. $q = x_l$ implies that optimists can and are willing to borrow as much as possible. Contradiction. $q = x_l(2 - \frac{x_l}{x_h})$ implies that for optimists, borrowing under loan $(1, \frac{q}{x_l})$ has the same return rate as selling CDS on loans $(R, \frac{q}{x_l})$ with $R \geq \frac{x_h}{x_l}$. For pessimists, buying CDS on loans $(R, \frac{q}{x_l})$ with $R \geq \frac{x_h}{x_l}$ gives the same return rate as lending under loan $(1, \frac{q}{x_l})$. Hence it is possible pessimists buy from optimists the credit default swaps on loans $(R, \frac{q}{x_l})$ with $R \geq \frac{x_h}{x_l}$. □

When $\lambda^o > \frac{x_h}{q}$ and $\lambda^p > 1$, the following lemma states that Ω should be like the one shown in Fig.12. In this case pessimists perceive it profitable to buy the credit default swap on a loan which they expect to go bad in the 2nd period while optimists perceive it profitable to buy the risky asset on margin under a different loan.

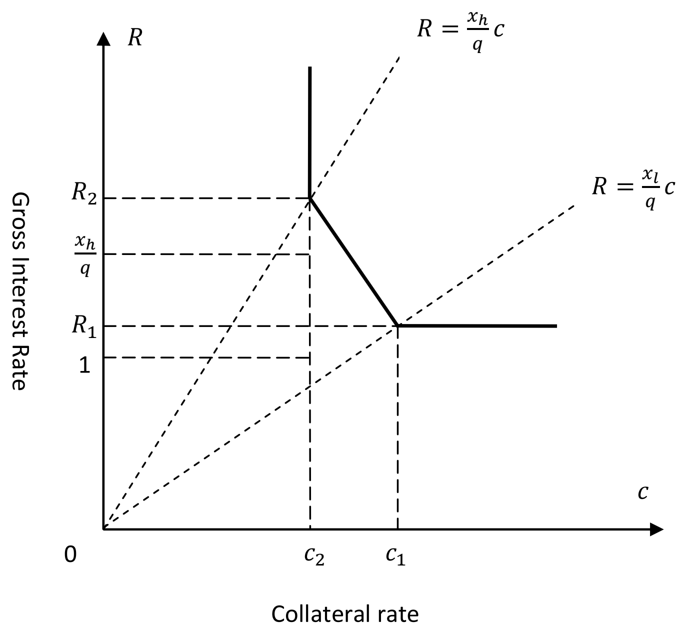


Fig. 12. Ω when credit default swaps are allowed and $\lambda^o > \frac{x_h}{q}$, $\lambda^p > 1$.

Lemma 16. *In equilibrium, if* $\lambda^o > \frac{x_h}{q}$ *and* $\lambda^p > 1$, $\min\{x_l(2 - \frac{x_l}{x_h}), \frac{x_h^2}{2x_h - x_l}\} < q < \max\{x_l(2 - \frac{x_l}{x_h}), \frac{x_h^2}{2x_h - x_l}\}$, *the only transactions are: optimists borrow from pessimists under the loan* (R_1, c_1) *and meanwhile pessimists buy from optimists the credit default* *swaps on loan* (R_2, c_2)*, where*

$$R_1 = \frac{x_l}{q} c_1 \; , \tag{3.36}$$

$$R_2 = \frac{x_h}{q} c_2 \; , \tag{3.37}$$

$$R_1 = \frac{R_2 - \frac{x_l}{q} c_2}{R_2 - 1} > 1 \; , \tag{3.38}$$

$$R_2 = \frac{\frac{x_h}{q} c_1 - R_1}{c_1 - 1} > \frac{x_h}{q} \; , \tag{3.39}$$

$$\frac{1-\theta}{\theta} \frac{a_0}{k_0} [1 - \frac{1}{R_2(1-\theta)}] - (1 - \frac{1}{c_1 \theta}) q = 0 \; , \tag{3.40}$$

and

$$\Omega = \{(R,c) | R > R_2 \; , \; c = c_2\} \cup \{(R,c) | R = R_1 \; , \; c > c_1\}$$
$$\cup \{(R,c) | (R,c) = \beta \cdot (R_1, c_1) + (1-\beta) \cdot (R_2, c_2) \; , \; \forall \beta \in [0,1]\} \; . \tag{3.41}$$

Proof. Step1: $\lambda^o > \frac{x_h}{q}$ and $\lambda^p > 1$ imply that optimists must borrow and pessimists must buy CDS. That optimists do not borrow under any loan implies that optimists do not hold the risky asset. Since $\lambda^p > 1$, that pessimists hold the risky asset implies that they borrow under certain loans. Suppose pessimists borrow under a loan $(R_0, c_0) \in \Omega$. It must be that $R_0 > 1$ since $\lambda^o > \frac{x_h}{q} \geq 1$, which implies $R_0 \geq \frac{x_l}{q} c_0$ and thus $c_0 = 1$. But $\frac{x_l}{q} \leq 1$. Contradiction. Hence no one holds the risky asset. Contradiction. That pessimists do not buy CDS on any loan implies that optimists do not hold the risk-free asset. Since $\lambda^p > 1$, that pessimists hold the risk-free asset implies that they sell CDS on certain loans. Suppose pessimists sell CDS under a loan $(R_0, c_0) \in \Omega$. It must be that $R_0 > 1$ and $\frac{x_l}{q} c_0 > 1$ since $\lambda^p > 1$, and $\frac{R_0 - \frac{x_h}{q} c_0}{R_0 - 1} > 1$ since $\lambda^o > \frac{x_h}{q} \geq 1$. Hence $\frac{x_h}{q} c_0 < 1$ contradicting with $\frac{x_l}{q} c_0 > 1$. Therefore no one holds the risk-free asset. Contradiction. Step2: Express R_2 in q by using equations

(3.36)-(3.39):

$$R_2 = \frac{(\frac{x_h}{x_l} - 1)(1 - \frac{x_l}{x_h}) - 1}{\frac{q}{x_l}(1 - \frac{x_l}{x_h}) - 1} \ .$$

Since $R_2 \in (\frac{x_h}{q}, \frac{x_h}{x_l})$, $\min\{x_l(2 - \frac{x_l}{x_h}), \frac{x_h^2}{2x_h - x_l}\} < q < \max\{x_l(2 - \frac{x_l}{x_h}), \frac{x_h^2}{2x_h - x_l}\}$. Step3: optimists borrow from pessimists under loan (R_1, c_1), it must be that $R_1 = \frac{x_l}{q} c_1$; pessimists buy from optimists CDS on loan (R_2, c_2), it must be that $R_2 = \frac{x_h}{q} c_2$. Step4: it is easy to check that when Ω satisfies (3.41), the only loan transaction is on loan (R_1, c_1) and the only transaction of CDS is on loan (R_2, c_2). $\square$

The following theorem presents how the equilibrium changes as θ increases. Fig. 13 illustrates how the price of the risky asset changes as θ increases when credit default swaps both are prohibited and are allowed, in the case where $\frac{x_h^2}{2x_h - x_l} > x_l(2 - \frac{x_l}{x_h})$.

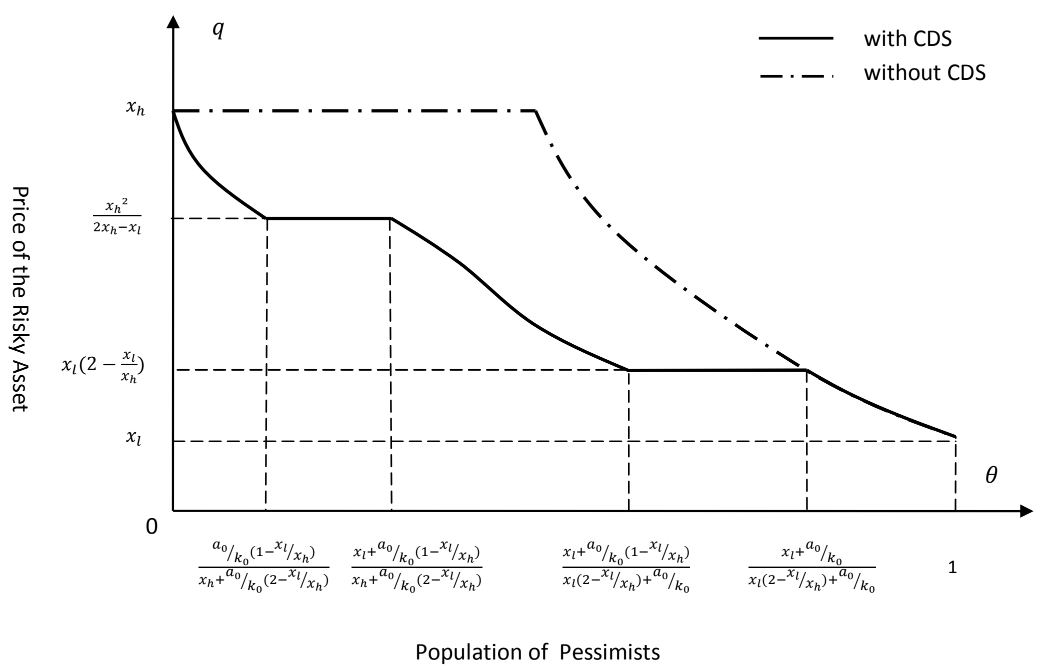


Fig. 13. q when credit default swaps are allowed and $\frac{x_h^2}{2x_h - x_l} > x_l(2 - \frac{x_l}{x_h})$.

Theorem 7. *When trading credit default swaps is allowed, the equilibrium changes as follows.*

- *If*

$$\theta > \frac{x_l + \frac{a_0}{k_0}}{x_l(2 - \frac{x_l}{x_h}) + \frac{a_0}{k_0}} \,, \tag{3.42}$$

$q = \frac{1-\theta}{\theta}\frac{a_0}{k_0} + \frac{x_l}{\theta}$, *optimists borrow from pessimists under the loan* $(1, \frac{q}{x_l})$, *there is no transaction in the market for credit default swaps, and* Ω *satisfies (3.34).*

- *If*

$$\frac{x_l + \frac{a_0}{k_0}(1 - \frac{x_l}{x_h})}{x_l(2 - \frac{x_l}{x_h}) + \frac{a_0}{k_0}} \leq \theta \leq \frac{x_l + \frac{a_0}{k_0}}{x_l(2 - \frac{x_l}{x_h}) + \frac{a_0}{k_0}} \,, \tag{3.43}$$

$q = x_l(2 - \frac{x_l}{x_h})$, *optimists borrow from pessimists under the loan* $(1, \frac{q}{x_l})$, *pessimists buy from optimists the credit default swaps on loans* $(R, \frac{q}{x_l})$ *with* $R \geq \frac{x_h}{x_l}$, *and* Ω *satisfies (3.34).*

- *If*

$$\theta < \frac{\frac{a_0}{k_0}(1 - \frac{x_l}{x_h})}{x_h + \frac{a_0}{k_0}(2 - \frac{x_l}{x_h})} \,, \tag{3.44}$$

$q = \frac{(1-\theta)a_0 x_h}{x_h \theta k_0 + a_0}$, *pessimists buy from optimists the credit default swaps on loan* $(\frac{x_h}{q}, 1)$, *there is no loan transaction, and* Ω *satisfies (3.35).*

- *If*

$$\frac{\frac{a_0}{k_0}(1 - \frac{x_l}{x_h})}{x_h + \frac{a_0}{k_0}(2 - \frac{x_l}{x_h})} \leq \theta \leq \frac{x_l + \frac{a_0}{k_0}(1 - \frac{x_l}{x_h})}{x_h + \frac{a_0}{k_0}(2 - \frac{x_l}{x_h})} \,, \tag{3.45}$$

$q = \frac{x_h^2}{2x_h - x_l}$, *pessimists buy from optimists the credit default swaps on loan* $(\frac{x_h}{q}, 1)$, *optimists borrow from pessimists under loans* $(\frac{x_h}{q}, c)$ *with* $c \geq \frac{x_h}{x_l}$, *and* Ω *satisfies (3.35).*

- *If*

$$\frac{x_l + \frac{a_0}{k_0}(1 - \frac{x_l}{x_h})}{x_h + \frac{a_0}{k_0}(2 - \frac{x_l}{x_h})} < \theta < \frac{x_l + \frac{a_0}{k_0}(1 - \frac{x_l}{x_h})}{x_l(2 - \frac{x_l}{x_h}) + \frac{a_0}{k_0}}\ , \tag{3.46}$$

  optimists borrow from pessimists under loan (R_1, c_1)*, pessimists buy from optimists the credit default swaps on loan* (R_2, c_2)*, such that (3.36)-(3.40) hold, and* Ω *satisfies (3.41).*

Two implications from the theorem deserve to be mentioned. First, as credit default swaps are introduced, the price of the risky asset falls in general. The intuitive reason is that pessimists use credit default swaps to bet against some loans backed by the risky asset. Pessimists believe the return from the risky asset to be low and hence they expect some loans to go bad for sure in the 2nd period. Meanwhile, optimists believe the return from the risky asset to be high and hence that these loans will be paid off. Selling the credit default swaps requires optimists to hold certain amount of the risk-free asset as the collateral, and as long as the spreads of the credit default swaps are high, optimists are directed from buying the risky asset on margin to selling the credit default swaps, which as a result dampens the demand for the risky asset in general.

Second, it is not always the case that the price of the risky asset falls as credit default swaps are introduced. When the population of pessimists is large enough, the introduction of credit default swaps in fact does nothing to the price of the risky asset and the market for collateralized loans. As Fig. 13 illustrates, this is when (3.42) holds. In this case, before credit default swaps are introduced, optimists perceive sizable profits of buying the risky asset on margin. After credit default swaps are introduced, no transactions of credit default swaps take place in equilibrium. This is not because there is a lack of incentive on the side of CDS buyers, but because optimists perceive that the return rate of buying the risky asset on margin dominates that of selling the credit default swaps and hence there is a lack of incentive on the side of CDS sellers. As in Fig. 11, pessimists perceive that the return rate of buying the credit default swap on loan $(\frac{x_h}{x_l}, \frac{q}{x_l})$ is 1, which is equal to that of lending under loan $(1, \frac{q}{x_l})$, while optimists perceive that the return rate of selling the credit default swap on loan $(\frac{x_h}{x_l}, \frac{q}{x_l})$ is $\frac{x_h}{x_l}$, which is less than the return rate of buying the risky asset on margin under loan $(1, \frac{q}{x_l})$.

CHAPTER IV

CONCLUSION

In the present study, I postulate that economic agents have heterogeneous beliefs, and I theoretically analyze collateralized loan transactions among economic agents arising from the different beliefs. And I make collateral requirements endogenously determined. Below, I discuss possible extensions for the two works.

A. Speculative currency crises

I show that loan transactions among domestic residents arising from heterogeneous beliefs make an exchange rate peg vulnerable even though heterogeneous beliefs per se bring stability. This result has an immediate policy implication: the central bank should try to curb private transactions which destabilize the system. Nevertheless, it is not obvious what instruments that the central bank should use to accomplish this goal.

An interest rate defense is a policy measure, which is often mentioned among policy-makers. It says that central banks should raise interest rates to fend off speculative attacks. The idea is that high interest rates discourage short sales against domestic money. Since in the standard models of currency crises the private sector is a homogeneous entity, if some one wants to sell domestic money short, no one willingly takes the counterpart position. In my model, due to heterogeneous beliefs, some domestic residents sell domestic money short and others willingly lend. Hence I am able to use the framework in the present study to examine the conventional wisdom on an interest rate defense.

B. Credit default swaps

I show that credit default swaps only as insurance have no effect on asset prices. The key assumption is that a sufficient amount of collateral must be posted to back up the promise made in a credit default swap so that the seller of the credit default swap would be able to deliver payoffs in all contingencies. This is obviously counter-factual in the light of the case of AIG. To relax this assumption and see how the equilibrium changes is a main direction to explore.

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APPENDIX A

PROOFS

Proof of Lemma 3

Proof. Suppose in an equilibrium domestic residents $\theta_x \leq f_t^g$ borrow from domestic residents $\theta_x > f_t^g$ at date t under the loan $(R, c_f, c_m) \in \Omega_t$. Step1: If $c_m = 0$, it must be that $c_f = 1$ and $R \geq \frac{1+i^*}{1-\bar{\pi}}$. Since $r(0; R, c_f, c_m) \geq r(\bar{\pi}; R, c_f, c_m)(1-\bar{\pi}) \geq (1+i^*)$, $c_f > 1$ implies $r(0; R, c_f, c_m) = 1 + i^*$ and $r(\bar{\pi}; R, c_f, c_m)(1-\bar{\pi}) = 1 + i^*$, which in combination, implies $R = \frac{1+i^*}{1-\bar{\pi}}$ and $R = 1 + i^*$. Contradiction. Step2: If $c_m > 0$, $c_f > 0$, and $R < c_f(1+i^*) + c_m$, $r(0; R, c_f, c_m) > r(\bar{\pi}; R, c_f, c_m)(1-\bar{\pi}) \geq (1 + i^*)$. Hence for domestic residents $\theta_x \leq f_t^g$, $\mu_t^f > 0$. As a result, this case can not happen in the equilibrium, intuitively because domestic residents $\theta \leq f_t^g$ as borrowers have incentives to ask domestic residents $\theta_x > f_t^g$ as lenders to lower the collateral requirements. Precisely, $\exists \epsilon > 0$, such that $R < (c_f - \epsilon)(1+i^*) + c_m$. To make domestic residents $\theta_x \leq f_t^g$ not prefer loan $(R, c_f - \epsilon, c_m)$ over $(R, c_f, c_m) \in \Omega_t$ in borrowing, it must be that $p_t(R, c_f - \epsilon, c_m) < 1$ due to the fact that for domestic residents $\theta_x \leq f_t^g$, $\mu_t^f > 0$. But domestic residents $\theta_x > f_t^g$ prefer $(R, c_f - \epsilon, c_m)$ over (R, c_f, c_m) in lending provided $p(R, c_f - \epsilon, c_m) < 1$ due to the fact that $r(\bar{\pi}; R, c_f - \epsilon, c_m)(1-\bar{\pi}) = r(\bar{\pi}; R, c_f, c_m)(1-\bar{\pi})$. Contradiction. Step3: The case $c_m > 0$, $c_f \geq 0$, $R \geq c_f(1+i^*)+c_m$ but $R(1-\bar{\pi}) < c_f(1+i^*)+c_m(1-\bar{\pi})$ can not happen in equilibrium as well, for the same reason as in Step2, i.e., intuitively because domestic residents $\theta_x \leq f_t^g$ as borrowers have incentives to ask domestic residents $\theta_x > f_t^g$ as lenders to lower the collateral requirements. Step4: The case $c_m > 0$, $c_f > 0$, $R(1-\bar{\pi}) \geq c_f(1+i^*)+c_m(1-\bar{\pi})$ can happen in the equilibrium only when $c_f(1+i^*)+c_m(1-\bar{\pi}) = 1 + i^*$ and thus $R \geq \frac{1+i^*}{1-\bar{\pi}}$. When $c_m > 0$ and $c_f > 0$, it must be that for domestic residents $\theta_x \leq f_t^g$, $\frac{\lambda_t}{\lambda_t - l'(m_t)} = \frac{1+i^*}{1-\bar{\pi}}$ and $\lambda_t = \frac{l'(m_t)c_m}{c_m+c_f-1}$, which in combination implies $c_f(1+i^*)+c_m(1-\bar{\pi}) = 1+i^*$. Hence for domestic residents $\theta_x > f_t^g$, $\lambda_t = 1$. Moreover, for domestic residents $\theta_x \leq f_t^g$, $\lambda_t = \frac{1}{1-\bar{\pi}}$. If for domestic residents $\theta_x \leq f_t^g$, $\lambda_t > \frac{1}{1-\bar{\pi}}$, they do not lend to domestic residents $\theta_x > f_t^g$, which implies domestic residents $\theta_x \leq f_t^g$ hold more domestic money than what is given by $l'(m_t) > \frac{i^*}{1+i^*}$. Contradiction. Step5: The case $c_m > 0$, $c_f = 0$, and $R < c_m$ can happen in equilibrium provided for borrowers $\mu_t^m = 0$. As a result, for domestic residents $\theta_x \leq f_t^g$, $\lambda_t = \frac{R}{1+i^*}$ and $R \geq \frac{1+i^*}{1-\bar{\pi}}$. $R > \frac{1+i^*}{1-\bar{\pi}}$ implies that for domestic residents $\theta_x > f_t^g$, $\lambda_t = R\frac{1-\bar{\pi}}{1+i^*} > 1$. Domestic residents $\theta_x \leq f_t^g$ do not lend under loans the return of which in the contingency of no collapse is less than R. Suppose $\exists$ some loan $(\tilde{R}, \tilde{c}_f, \tilde{c}_m) \in \Omega_t$ such that $r(0; \tilde{R}, \tilde{c}_f, \tilde{c}_m) \geq R$. To domestic residents $\theta_x > f_t^g$ want to borrow under $(\tilde{R}, \tilde{c}_f, \tilde{c}_m)$, it must be that $r(\bar{\pi}; \tilde{R}, \tilde{c}_f, \tilde{c}_m)(1-\bar{\pi}) = R(1-\bar{\pi})$, which further implies $\tilde{c}_f = 0$ and domestic residents $\theta_x > f_t^g$, $\mu_t^m = 0$. The aggregate real cash balances at date t is $K(f_t^g)\mathcal{L}(R-1) + (1-K(f_t^g))\mathcal{L}(i^*+\bar{\pi}) < K(f_t^g)\mathcal{L}(i^*) + (1-K(f_t^g))\mathcal{L}(i^*+\bar{\pi})$, which contradicts with the fact the public as a whole do not hold the foreign bond and do not consume at date t. Step6: When $c_m > 0$, $c_f = 0$, and $R \geq c_m$ it must be that $c_m = \frac{1+i^*}{1-\bar{\pi}}$. If $c_m > \frac{1+i^*}{1-\bar{\pi}}$, since for domestic residents $\theta \leq f_t^g$, $l'(m_t) \geq \frac{c_m - 1}{1+i^*}$, and then for domestic residents $\theta_x \leq f_t^g$, $\lambda_t > \frac{1}{1-\bar{\pi}}$. Contradiction as the same as in Step5. □

Proof of Lemma 4

Proof. Step1: The case $R > c_f(1+i^*)+c_m$ and $c_f > 0$ can not happen in equilibrium, intuitively because domestic residents $\theta_x > f_t^g$ as borrowers have incentives to ask domestic residents $\theta_x \leq f_t^g$ as lenders to lower the promised interest rate. Formally, $\exists \epsilon > 0$ such that $R - \epsilon > c_f(1+i^*) + c_m$ and $(R-\epsilon)(1-\bar{\pi}) < c_f(1+i^*) + c_m(1-\bar{\pi})$ provided $c_f > 0$. To make domestic residents $\theta_x > f_t^g$ not prefer loan $(R-\epsilon, c_f, c_m)$

over loan $(R, c_f, c_m) \in \Omega_t$ in borrowing, $p_t(R-\epsilon, c_f, c_m) < 1$. But domestic residents $\theta_x \leq f_t^g$ prefer loan $(R-\epsilon, c_f, c_m)$ over loan (R, c_f, c_m) provided $p_t(R-\epsilon, c_f, c_m) < 1$. Contradiction. Step2: The case that for domestic residents $\theta_x > f_t^g$, $\mu_t^f > 0$ and $\mu_t^m = 0$ can not happen in equilibrium. If $R \leq c_f(1+i^*) + c_m$, $c_f = 0$ due to $\mu_t^f > 0$ and $\mu_t^m = 0$. If domestic residents $\theta_x > f_t^g$ hold the foreign bond, they must borrow under some loan $(\tilde{R}, \tilde{c}_f, \tilde{c}_m) \in \Omega_t$ where $\tilde{c}_f > 0$, which is impossible by Step1. Hence domestic residents $\theta_x > f_t^g$ do not hold the foreign bond, do not consume, and moreover do not lend due to $\mu_t^f > 0$ by Lemma 3 and thus $\lambda_t > 1$, which contradicts with the fact that for domestic residents $\theta_x > f_t^g$, $l'(m_t) > \frac{i^*+\pi}{1+i^*}$. Step3: The case that for domestic residents $\theta_x > f_t^g$, $\mu_t^f = 0$ and $\mu_t^m > 0$ can not happen in equilibrium. If $R \leq c_f(1+i^*) + c_m$, $c_m = 0$ due to $\mu_t^f = 0$ and $\mu_t^m > 0$. Hence domestic residents $\theta_x > f_t^g$ must borrow under some loan $(\tilde{R}, 0, \tilde{c}_m) \in \Omega_t$, where $\tilde{R} > \tilde{c}_m$ by Step1. For domestic residents $\theta_x > f_t^g$, $\lambda = 1$ and thus $\tilde{c}_m < \frac{1+i^*}{1-\bar{\pi}}$ due to $\mu_m > 0$. Since $(\frac{1+i^*}{1-\bar{\pi}}, \frac{1}{1-\bar{\pi}}, 0) \in \Omega_t$, the loan $(\tilde{R}, 0, \tilde{c}_m)$ are not preferred by domestic residents $\theta_x \leq f_t^g$. Step4: In the case that for domestic residents $\theta_x > f_t^g$, $\mu_t^f = 0$ and $\mu_t^m = 0$, either $\frac{1+i^*}{1-\bar{\pi}} = R \leq c_f(1+i^*) + c_m$, or $c_f = 0$ and $R > c_m = \frac{1+i^*}{1-\bar{\pi}}$. Step5: In the case for domestic residents $\theta_x > f_t^g$, $\mu_t^f, \mu_t^m > 0$, either $R = c_f(1+i^*) + c_m$, or $c_f = 0$ and $R > c_m$. When $R = c_f(1+i^*) + c_m$, if $c_f, c_m > 0$, and then for domestic residents $\theta_x > f_t^g$, $\mu_t^f = \mu_t^m(1+i^*)$ and $R < \frac{1+i^*}{1-\pi}$. When $R = c_f(1+i^*) + c_m$, if $c_f > 0$ but $c_m = 0$, and then $c_f < \frac{1}{1-\bar{\pi}}$ and $R < \frac{1+i^*}{1-\bar{\pi}}$. When $c_f = 0$ and $R \geq c_m$, domestic residents $\theta_x > f_t^g$ do hold the foreign bond and thus must borrow under some loan $(\tilde{R}, \tilde{c}_f, \tilde{c}_m) \in \Omega_t$ such that $\tilde{R} = \tilde{c}_f(1+i^*) + \tilde{c}_m$ and $\tilde{c}_f > 0$. As a result, $c_m = \tilde{R} < \frac{1+i^*}{1-\bar{\pi}}$. When $R = c_f(1+i^*)$ and $c_m = 0$, for domestic residents $\theta_x > f_t^g$, $\mu_t^f = \mu_t^m(1+i^*)$. Suppose $\mu_t^f < \mu_t^m(1+i^*)$. Domestic residents $\theta_x > f_t^g$ must borrow under some loan $(\tilde{R}, 0, \tilde{c}_m) \in \Omega$ where $\tilde{R} \geq \tilde{c}_m = R = c_f(1+i^*)$, which contradicts with $\mu_t^f < \mu_t^m(1+i^*)$. $\square$

Proof of Lemma 6

Proof. Let $\tilde{L}(\cdot)|_{\Omega\sim(R_0,c_0)} = L(\cdot)|_{\Omega\sim(R_0,c_0)}$, and $\tilde{S}(\cdot)|_{\Omega\sim(R_0,c_0)} = S(\cdot)|_{\Omega\sim(R_0,c_0)}$. Step1: $L(R_0,c_0) > 0$ and $S(R_0,c_0) > 0$. Let $\tilde{L}(R_0,c_0) = \max\{L(R_0,c_0) - S(R_0,c_0), 0\}$, $\tilde{S}(R_0,c_0) = \max\{S(R_0,c_0) - L(R_0,c_0), 0\}$, $\tilde{k} = k$, and $\tilde{a} = a + R_0 \cdot \min\{S(R_0,c_0), L(R_0,c_0)\}$. $\{\tilde{k}, \tilde{a}, \tilde{L}(\cdot), \tilde{S}(\cdot)\}$ is feasible and generates the same amount of consumption good in the 2nd period as $\{k, a, L(\cdot), S(\cdot)\}$ for any given value of x. Step2: $L(R_0,c_0) > 0$ and $S(R_0,c_0) < 0$. Let $\tilde{L}(R_0,c_0) = L(R_0,c_0) - S(R_0,c_0)$, $\tilde{S}(R_0,c_0) = 0$, $\tilde{k} = k$, and $\tilde{a} = a + R_0 S(R_0,c_0)$. Step3: $L(R_0,c_0) < 0$ and $S(R_0,c_0) < 0$. Let $\tilde{L}(R_0,c_0) = \min\{L(R_0,c_0) - S(R_0,c_0), 0\}$, $\tilde{S}(R_0,c_0) = \min\{S(R_0,c_0) - L(R_0,c_0), 0\}$, $\tilde{k} = k$, and $\tilde{a} = a + R_0 \cdot \max\{S(R_0,c_0), L(R_0,c_0)\}$. Step4: $L(R_0,c_0) < 0$ and $S(R_0,c_0) > 0$. Suppose $R_0 > \frac{x}{q}c_0$. $L(R_0,c_0) < 0$ implies $c_0 = 1$. Let $\tilde{L}(R_0,c_0) = 0$, $\tilde{S}(R_0,c_0) = S(R_0,c_0)$, $q\tilde{k} = qk + L(R_0,c_0)$ and $\tilde{a} = a$. Suppose $R_0 \leq \frac{x}{q}c_0$. $S(R_0,c_0) > 0$ implies $R_0 = 1$. Let $\tilde{L}(R_0,c_0) = L(R_0,c_0)$, $\tilde{S}(R_0,c_0) = 0$, $\tilde{k} = k$ and $\tilde{a} = a$. $\square$

Proof of Theorem 5

Proof. One direction: suppose $\{q^*, p^*(\cdot)\}$ is an equilibrium when credit default swaps are not allowed and $\{k^i, a^i, L^i(\cdot), 0\}$ denotes the corresponding choice by individual $i \in [0,1]$ in the equilibrium. The claim is sufficient to prove this direction that when the price of the risky asset is q^*, the price system for the collateralized loans is $p^*(\cdot)$, and credit default swaps are allowed but the uses of them are restricted under Assumption 3, for individual i, $\{k^i, a^i, L^i(\cdot), 0\}$ maximizes (3.9) subject to (3.11)-(3.13). Prove by contradiction. Suppose $\exists\{\tilde{k^i}, \tilde{a^i}, \tilde{L^i}(\cdot), \tilde{S^i}(\cdot)\}$ which gives more consumption good in the 2nd period than $\{k^i, a^i, L^i(\cdot), 0\}$. Due to the proof in Lemma 6, let $\forall(R,c), p^*(R,c) = 1$, $\tilde{S^i}(R,c) = 0$. Contradiction. The opposite direction: suppose $\{q^*, p^*(\cdot)\}$ is an equilibrium when credit default swaps are allowed but the uses of them are restricted under Assumption 3 and $\{k^i, a^i, L^i(\cdot), S^i(\cdot)\}$ denotes the corresponding choice by individual $i \in [0,1]$ in the equilibrium. Due to the proof in Lemma 6, I transform all individuals' choices by zeroing the transactions of CDS, i.e., $\exists\{\tilde{k^i}, \tilde{a^i}, \tilde{L^i}(\cdot), \tilde{S^i}(\cdot)\}$ which is a maximizer too. And $\forall(R,c), p^*(R,c) = 1$, $\tilde{k^i} = k^i$, $\tilde{a^i} = a^i + R\cdot S^i(R,c)$, $\tilde{L^i}(R,c) = L^i(R,c) - S^i(R,c)$, $\tilde{S^i}(R,c) = 0$. I show that after the transformations, all markets are cleared, i.e.,$\forall(R,c), p^*(R,c) = 1$, $\int_0^1 \tilde{L^i}(R,c)\,di \ = \ 0$, and $\int_0^1 \ \tilde{a^i}\ di \ = \ \int_0^1 \ a^i\ di \ = \ a_0$. □

Proof of Theorem 7

Proof. Step1: when $\lambda^o > \frac{x_h}{q}$ and $\lambda^p = 1$,

$$\theta \geq \frac{x_l + \frac{a_0}{k_0}(1 - \frac{x_l}{x_h})}{x_l(2 - \frac{x_l}{x_h}) + \frac{a_0}{k_0}}\ . \tag{A.1}$$

Step2: when $\lambda^o = \frac{x_h}{q}$ and $\lambda^p > 1$,

$$\theta \leq \frac{x_l + \frac{a_0}{k_0}(1 - \frac{x_l}{x_h})}{x_h + \frac{a_0}{k_0}(2 - \frac{x_l}{x_h})}\ . \tag{A.2}$$

Step3: when $\lambda^o > \frac{x_h}{q}$ and $\lambda^p > 1$ and $\frac{x_h^2}{2x_h - x_l} > x_l(2 - \frac{x_l}{x_h})$, R_2 is decreasing in q, and thus the left hand side of equation (3.40) is decreasing in q for any θ and is always decreasing in θ for any q. For θ satisfying (A.2), equation (3.40) holds provided $q > \frac{x_h^2}{2x_h - x_l}$. Contradiction. For θ satisfying (A.1), equation (3.40) holds provided $q < x_l(2 - \frac{x_l}{x_h})$. Contradiction. Hence (3.46) holds. Step4: when $\lambda^o > \frac{x_h}{q}$ and $\lambda^p > 1$ and $\frac{x_h^2}{2x_h - x_l} = x_l(2 - \frac{x_l}{x_h})$, $q = x_l(2 - \frac{x_l}{x_h})$. For θ satisfying (A.2), equation (3.40) holds provided $R_2 < \frac{x_h}{q}$. Contradiction. Forθ satisfying (A.1), equation (3.40) holds provided $R_2 > \frac{x_h}{x_l}$. Contradiction. Hence (3.46) holds. Step5: when $\lambda^o > \frac{x_h}{q}$ and $\lambda^p > 1$ and $\frac{x_h^2}{2x_h - x_l} < x_l(2 - \frac{x_l}{x_h})$, the left hand side of equation (3.40) is always monotonic in q given θ and is always decreasing in θ. For θ satisfying (A.2), if the left hand side of equation (3.40) is decreasing in q, equation (3.40) holds provided

$q > x_l(2-\frac{x_l}{x_h})$; if the left hand side of equation (3.40) is increasing in q, equation (3.40) holds provided $q < \frac{x_h^2}{2x_h - x_l}$. Contradiction. For θ satisfying (A.1), if the left hand side of equation (3.40) is decreasing in q, equation (3.40) holds provided $q < \frac{x_h^2}{2x_h - x_l}$; if the left hand side of equation (3.40) is increasing in q, equation (3.40) holds provided $q > x_l(2 - \frac{x_l}{x_h})$. Contradiction. $\square$

VITA

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